UNIT-1

UNIT: 1

Ques 1: State Wien's displacement law, Rayleigh-Jean's law. Write the assumptions of Planck's hypothesis.

Ans: **Wien's displacement law:-** Wien's observed that there is a wavelength at which radiation has maximum intensity at given temperature, which is given by

$$\boxed{\lambda_{max} \propto \frac{1}{T}} \quad \text{or} \quad \boxed{\lambda_{max} T = b} \text{ (constant)}$$

[b = 0.0029 m-K → Wien's constant]

Rayleigh-Jean's law:-
According to Rayleigh Jean's, the blackbody spectral energy distribution is

$$\boxed{E_\lambda d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda}$$

[k = Boltzmann constant = 1.38×10^{-23} J/K]

→ **Basic assumptions of Planck's hypothesis:-**
There are two assumptions:-

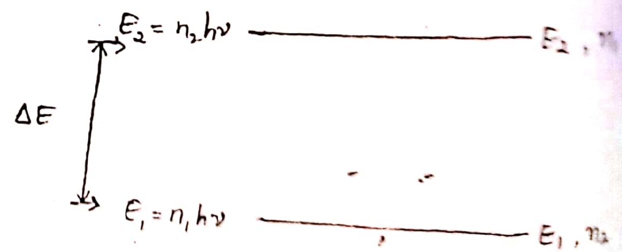
(1) The energy of oscillators have integral multiple of $h\nu$

$$\boxed{E_n = nh\nu}$$

(2) The energy difference between two states (initial and final state) is equal to difference of their energy.

$$E_2 - E_1 = (n_2 - n_1) h\nu$$

$$\boxed{\Delta E = \Delta n h\nu}$$



$\therefore \quad u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} e^{-hc/\lambda kT} d\lambda$

or $\boxed{u_\lambda d\lambda = \frac{C_1}{\lambda^5} e^{-C_2/\lambda kT} d\lambda}$ $\left[\begin{array}{l} 8\pi hc = C_1 \\ hc/k = C_2 \end{array}\right.$

↳ Valid for shorter wavelength region]

Case II: Rayleigh-Jeans law from Planck's law:
for long wavelength or λ is large, $e^{hc/\lambda kT} \approx 1 + \frac{hc}{\lambda kT}$

(neglect higher power of λ)

$\therefore$ Eq^n (5) becomes

$$u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{\left[1 + \frac{hc}{\lambda kT} - 1\right]}$$

or $\boxed{u_\lambda d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda}$

↳ Valid for longer wavelength region

Ques 2: What is the concept of de-Broglie matter waves. Why matter waves are associated with a particle generated when only it is in motion?

Ans: The wave associated with a moving material particle is called matter wave or de-Broglie waves.

According to de-Broglie concept, a moving particle always have a wave associated with it. If a particle of mass 'm' has momentum 'p' and 'λ' is the wavelength of wave associated with it, then according to de-Broglie

$\boxed{\lambda = h/p = \frac{h}{mv}}$ → for materialistic particle — (1)

Now, if $v = 0$, then by eq^n (1), $\boxed{\lambda = \infty}$, i.e, wave becomes indeterminate or we cannot define the wave nature of particle. and if $v = \infty$ then $\lambda = 0$. This shows that waves are generated by the motion of particles.

Ques 3: (a) Give characteristics and physical interpretation of wave function.

Ans: Physical interpretation :- (1) Wave function 'ψ' is complex function and has no physical significance itself.

(2) $|\psi|^2$ or $\psi^*\psi$ associated with a moving particle at a particular point (x, y, z) → gives probability of finding the particle at the point.

(3) $\boxed{\iiint |\psi|^2 dx\,dy\,dz = 1}$ → Normalized condition where 'ψ' is normalized function.

Characteristics :- (1) It must be normalized.
(2) It must be finite everywhere.
(3) It must be single valued.
(4) It must be continuous and its first derivative also be continuous.

(b) Show that $\psi(x,y,z,t) = \psi(x,y,z) e^{-i\omega t}$ is function of stationary state.

Ans:
$$\psi(x,y,z,t) = \psi(x,y,z) e^{-i\omega t}$$
$$|\psi(x,y,z,t)|^2 = \psi^*(x,y,z,t)\psi(x,y,z,t) = \psi^*(x,y,z) e^{i\omega t} \psi(x,y,z) e^{-i\omega t}$$
$$|\psi(x,y,z,t)|^2 = \psi^*(x,y,z)\psi(x,y,z)$$
$$|\psi(x,y,z,t)|^2 = |\psi(x,y,z)|^2 \quad \text{——— (1)}$$

Since, $|\psi|^2$ is not a function of time (from eq^n (1)), $\therefore$ $\psi(x,y,z,t) = \psi(x,y,z)e^{-i\omega t}$ is a function of stationary state.

Ques 4: Obtain time dependent and time independent Schrodinger wave equation.

Answer: Schrodinger time dependent wave equation:

Wave equation in 1-dimensional is given by

$$\frac{\partial^2 \psi}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} \quad \text{------} \quad (1)$$

General solution of eq^n (1) is given as,

$$\psi(x,t) = A e^{-i\omega[t - x/v]} \quad \text{------} \quad (2)$$

$\because$ $\omega = 2\pi\nu$ and $v = \nu\lambda$. So, eq (2) can be written as

$$\psi(x,t) = A e^{-2\pi i[\nu t - \frac{x}{\lambda}]} \quad \text{------} \quad (3)$$

Also, $E = h\nu$ and $\lambda = h/p$

$$\therefore \quad \psi(x,t) = A e^{-\frac{2\pi i}{h}[Et - px]} \quad \text{------} \quad (4)$$

Now, Differentiate eq^n (4) twice with respect to x,

$$\frac{\partial^2 \psi(x,t)}{\partial x^2} = A e^{-\frac{2\pi i}{h}[Et-px]} \times \left(+\frac{2\pi i p}{h}\right)^2$$

$$\frac{\partial^2 \psi(x,t)}{\partial x^2} = -\frac{4\pi^2 p^2 \psi}{h^2} \Rightarrow \boxed{p^2 = -\frac{1}{\psi}\frac{h^2}{4\pi^2}\frac{\partial^2 \psi(x,t)}{\partial x^2}} \quad \text{------} \quad (5)$$

Now, Differentiate eq^n (4) with respect to t,

$$\frac{\partial \psi(x,t)}{\partial t} = A e^{-\frac{2\pi i}{h}[Et-px]} \times \left(-\frac{2\pi i E}{h}\right)$$

$$\frac{\partial \psi(x,t)}{\partial t} = -\frac{2\pi i E}{h}\psi$$

$$\text{or} \quad E = -\frac{1}{\psi}\frac{h}{2\pi i}\frac{\partial \psi}{\partial t} = \frac{1}{\psi}\frac{ih}{2\pi}\frac{\partial \psi(x,t)}{\partial t} \quad \text{------} \quad (6)$$

If E and V be the total and potential energy of particle respectively, then its kinetic energy '$\frac{1}{2}mv^2$' is given by

$$E = \frac{1}{2}mv^2 + V$$

$$\text{or} \quad E = \frac{p^2}{2m} + V \quad \text{------} \quad (7)$$

Using eq^n (5) and (6) into (7),

$$\frac{1}{\psi}\frac{ih}{2\pi}\frac{\partial \psi}{\partial t} = -\frac{1}{\psi}\frac{h^2}{4\pi^2 2m}\frac{\partial^2 \psi}{\partial x^2} + V,$$

$$\text{or} \quad -\frac{h^2}{8\pi^2 m}\frac{\partial^2 \psi}{\partial x^2} + V\psi = \frac{ih}{2\pi}\frac{\partial \psi}{\partial t}$$

$\because$ $\hbar = h/2\pi$, we get

$$\boxed{-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi = i\hbar\frac{\partial \psi}{\partial t}} \quad \text{------} \quad (8)$$

Required Schrodinger time dependent eq^n in 1-dimension.

In 3-D, eq^n can be given as,

$$\boxed{\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = i\hbar\frac{\partial \psi}{\partial t}}$$

$$\text{or} \quad \boxed{H\psi = E\psi}$$

where $H = -\frac{\hbar^2}{2m}\nabla^2 + V$ → Hamiltonian Op

$E = i\hbar\frac{\partial}{\partial t}$ → Energy Operator.

Schrodinger time independent Equation:

Wave equation in 1-dimension is given as

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} \quad \text{—— (1)}$$

General solution of eq (1) is given by

$$\psi(x,t) = A e^{-i\omega[t - \frac{x}{v}]} \quad \text{—— (2)}$$

$\because$ $\omega = 2\pi\nu$ and $v = \nu\lambda$, So, eq (2) as

$$\psi(x,t) = A e^{-2\pi i[\nu t - \frac{x}{\lambda}]} \quad \text{—— (3)}$$

Also, $E = h\nu$ and $\lambda = h/p$, So, eq^n (3) as

$$\psi(x,t) = A e^{-\frac{2\pi i}{h}[Et - px]} \quad \text{—— (4)}$$

Wave function can be separated as time dependent and space dependent parts,

$$\therefore \psi(x,t) = A e^{-\frac{2\pi i}{h}Et} e^{\frac{2\pi i}{h}px}$$

If $\psi(x) = \psi_0 = A e^{+\frac{2\pi i px}{h}}$

$$\therefore \psi(x,t) = \psi_0 e^{-\frac{2\pi i Et}{h}} \quad \text{—— (5)}$$

Now, differentiate eq^n (5) twice with respect to x,

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{\partial^2 \psi_0}{\partial x^2} e^{-\frac{2\pi i Et}{h}} \quad \text{—— (6)}$$

Differentiate eq^n (5) with respect to 't', once,

$$\frac{\partial \psi}{\partial t} = \psi_0 e^{-\frac{2\pi i Et}{h}} \times \left(-\frac{2\pi i E}{h}\right) \quad \text{—— (7)}$$

Using Schrodinger time dependent equation in 1-D, i.e,

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi = i\hbar\frac{\partial \psi}{\partial t} \quad \text{—— (8)}$$

Using equation (5), (6) and (7) in (8)

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi_0}{\partial x^2} e^{-\frac{2\pi i Et}{h}} + V\psi_0 e^{-\frac{2\pi i Et}{h}} = i\hbar\psi_0 e^{-\frac{2\pi i Et}{h}} \times \left(-\frac{2\pi i E}{h}\right)$$

or

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi_0}{\partial x^2} + V\psi_0 = i\hbar\psi_0\left(-\frac{2\pi i E}{h}\right)$$

or

$$\boxed{-\frac{\hbar^2}{2m}\frac{\partial^2 \psi_0}{\partial x^2} + V\psi_0 = E\psi_0} \quad \text{—— (9)}$$

or

$$\boxed{\frac{\partial^2 \psi_0}{\partial x^2} + \frac{2m}{\hbar^2}(E - V)\psi_0 = 0}$$

Required time independent Schrodinger equation.

In General,

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0}$$

In 3-D,

$$\boxed{\nabla^2\psi + \frac{2m}{\hbar^2}(E - V)\psi = 0}$$

Note :- For free particle, $V = 0$

$$\therefore \boxed{\nabla^2\psi + \frac{2m}{\hbar^2}E\psi = 0} \rightarrow$$ Schrodinger wave e[quation] for free particle.

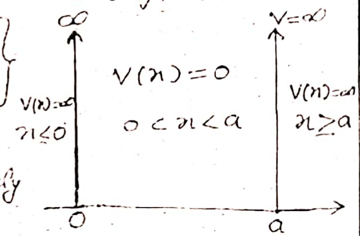
Ques. Find an expression for the energy states and eigen function for one dimensional box problem.

Ans. Particle in one-dimensional box:-

Let us consider the case of a particle of mass m moving along x axis between two rigid walls A and B at $x=0$ and $x=a$. The particle is free to move between the walls.

The potential function is defined as

$$\left.\begin{aligned} V(x) &= 0 \quad \text{for } 0<x<a \\ V(x) &= \infty \quad \text{for } 0 \geq x \geq a \end{aligned}\right\}$$



Under this condition, particle is said to move in an infinitely deep potential well or infinite square well.

The Schrodinger wave eq^n for the particle within the box $(V=0)$ is

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}E\psi = 0 \quad \text{——— (1)}$$

or

$$\frac{\partial^2\psi}{\partial x^2} + K^2\psi = 0 \quad \text{——— (2)}$$

where

$$K^2 = \frac{2mE}{\hbar^2} \quad \text{——— (3)}$$

The general solution of eq^n (2) is given as

$$\psi = A\sin Kx + B\cos Kx \quad \text{——— (4)}$$

Apply the boundary condition, $\psi=0$ at $x=0$ and $x=a$, to eq^n (4)

$$\therefore \quad 0 = A\sin K\cdot 0 + B\cos K\cdot 0$$
$$\Rightarrow \quad B=0$$

again at $x=a$, $\psi=0$,

$$\therefore \quad A\sin ka = 0$$

$$\boxed{ka = \pm n\pi} \quad \text{——— (5)}$$

where $n=1,2,3\ldots$

$n\neq 0$, because for $n=0$, $k=0$

using eq^n(3) and eq^n(5), we get

$$E_n = \frac{n^2\hbar^2\pi^2}{2ma^2}$$

$$\boxed{E_n = \frac{n^2h^2}{8ma^2}} \quad \text{——— (6)} \quad \left[\because \hbar = \frac{h}{2\pi}\right]$$

From eq^n(6), it is clear that particle can not have an arbitrary energy, but can have certain discrete energy corresponding to $n=1,2,3\ldots$ Each permitted energy is called eigen value of the particle and constitute the energy level of the system. The corresponding eigen function is given by

$$\psi = A\sin kx$$

Applying normalisation condition for finding the value of A,

$$\int_0^a |\psi_n|^2 dx = 1$$

$$A^2\int_0^a \sin^2\frac{n\pi x}{a}\,dx = 1$$

$$A^2\int_0^a \frac{1}{2}\left\{1-\cos\left(\frac{2n\pi x}{a}\right)\right\}dx = 1$$

$$\frac{A^2}{2}\left[x - \frac{\sin\left(\frac{2n\pi x}{a}\right)}{\frac{2n\pi}{a}}\right]_0^a = 1$$

$$\left[\because \sin^2\theta = \frac{1-\cos 2\theta}{2}\right]$$

$$\frac{A^2}{2}(a) = 1$$

$$\Rightarrow A = \sqrt{\frac{2}{a}}$$

$\because$ The normalized wavefunction for n^{th} state is given

$$\boxed{\psi_n = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)}$$

→ Required expression for eigen wave-function

Ques 5(b) Show that energy levels are not equally spaced for the particle in one dimensional Box.

Ans. We know that energy for a particle travelling in one-dimensional box is given by

$$E_n = \frac{n^2h^2}{8ma^2}$$

Putting $n = 1, 2, 3, 4, 5$ — so on, we get

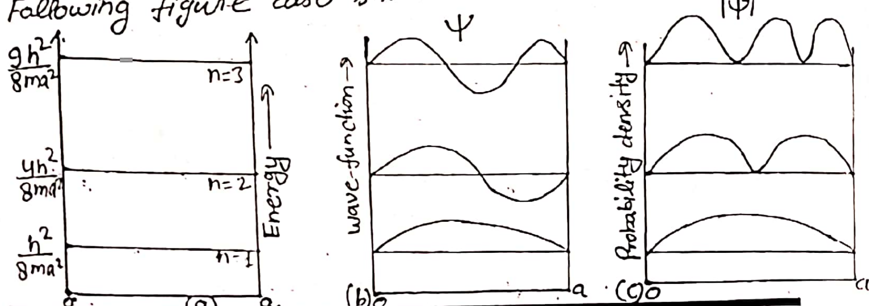
$$E_n = \frac{n^2h^2}{8ma^2}; \quad E_1 = \frac{h^2}{8ma^2}; \quad E_2 = \frac{4h^2}{8ma^2} = 4E_1$$

$$E_3 = \frac{9h^2}{8ma^2} = 9E_1, \quad E_4 = \frac{16h^2}{8ma^2} = 16E_1$$

Now, the difference betn energy in various states

$$E_2 - E_1 = 3E_1; \quad E_3 - E_2 = 5E_1; \quad E_4 - E_3 = 7E_1$$

So energy for a particle in 1-dimension box are not equally spaced from above eqn. Following figure also shows the same.



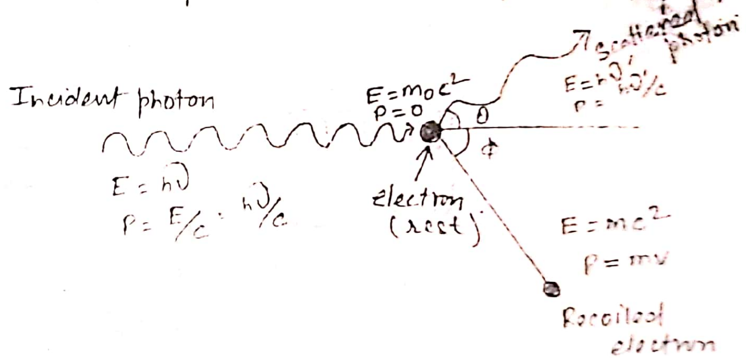
Question 6 :→ What is compton effect? Derive a suitable expression for compton shift $\lambda' - \lambda = \frac{h}{m_0c}(1 - \cos\theta)$.

Compton effect :→

When a monochromatic beam of high frequency radiation is scattered by electrons, the scattered radiation contain the lower frequency or greater wavelength along with radiation of unchanged frequency or wavelength.

⇒ The radiation of unchanged wavelength/frequency called unmodified radiations.

This phenomenon is called the Compton Effect.



The energy of system before collision $= h\nu + m_0c^2$

The energy of system after collision $= h\nu' + mc^2$

According to law of conservation of energy

$$h\nu + m_0c^2 = h\nu' + mc^2$$

$$mc^2 = (h\nu - h\nu') + m_0c^2 \quad \text{--- (1)}$$

According to law of conservation of momentum (along x axis & perpendicular)

Along x axis →

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\theta + mv\cos\phi$$

$$mvc\cos\phi = h\nu - h\nu'\cos\theta \quad \text{——} (2)$$

Along y axis →

$$0 + 0 = \frac{h\nu'}{c}\sin\theta - mv\sin\phi$$

$$mvc\sin\phi = h\nu'\sin\theta \quad \text{——} (3)$$

Squaring eqn (2) & (3), then adding,

$$m^2v^2c^2\cos^2\phi + m^2v^2c^2\sin^2\phi = (h\nu - h\nu'\cos\theta)^2 + (h\nu'\sin\theta)^2$$

$$m^2v^2c^2 = (h\nu)^2 + (h\nu'\cos\theta)^2 - 2(h\nu)(h\nu')\cos\theta + (h\nu'\sin\theta)^2$$

$$m^2v^2c^2 = (h\nu)^2 + (h\nu')^2 - 2(h\nu)(h\nu')\cos\theta \quad \text{——} (4)$$

Squaring equation (1)

$$m^2c^4 = m_0^2c^4 + (h\nu)^2 + (h\nu')^2 - 2(h\nu)(h\nu') + 2m_0c^2(h\nu - h\nu') \quad \text{——} (5)$$

Subtracting (4) from 5, we get

$$m^2c^4 - m^2v^2c^2 = m_0^2c^4 + 2(h\nu)(h\nu')(\cos\theta - 1) + 2m_0c^2(h\nu - h\nu') \quad \text{——} (6)$$

According to the theory of relativity

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} \quad \text{or} \quad m^2 = \frac{m_0^2}{(1 - v^2/c^2)}$$

$$m^2(1 - v^2/c^2) = m_0^2$$

$$m^2(c^2 - v^2) = m_0^2c^2$$

multiplying both sides by c^2

$$m^2(c^2 - v^2)c^2 = m_0^2c^2 \times c^2$$

$$m^2(c^2 - v^2)c^2 = m_0^2c^4 \quad \text{——} (7)$$

From eqn (6) & (7)

$$\cancel{m_0^2c^4} = \cancel{m_0^2c^4} + 2(h\nu)(h\nu')(\cos\theta - 1) + 2m_0c^2(h\nu - h\nu')$$

$$0 = 2(h\nu)(h\nu')(\cos\theta - 1) + 2m_0c^2(h\nu - h\nu')$$

$$2(h\nu)(h\nu')(1 - \cos\theta) = 2m_0c^2(h\nu - h\nu')$$

$$\frac{\nu - \nu'}{\nu\nu'} = \frac{h}{m_0c^2}(1 - \cos\theta)$$

$$\boxed{\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0c^2}(1 - \cos\theta)} \quad \text{——} (8)$$

we know $\nu = \frac{c}{\lambda}$, $\nu' = \frac{c}{\lambda'}$

$$\boxed{\lambda' - \lambda = \frac{h}{m_0c}(1 - \cos\theta)} \quad \text{——} (9)$$

Equation (9) represent the expression for Compton shift.

Cases:→ → When $\theta = 0$, $\cos 0 = 1$

$$\Delta\lambda = \lambda' - \lambda = 0$$

→ When $\theta = 90°$, $\cos 90 = 0$

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0c}$$

$$\Delta\lambda = \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}} = 0.0243\,\text{Å}$$

→ When $\theta = 180°$, $\cos 180° = -1$

$$\Delta\lambda = \lambda' - \lambda = \frac{2h}{2m_0c} = 0.04652\,\text{Å}$$

Question 7(a):→ Calculate the energy of oscillator of frequency 4.2×10^{12} Hz at 27°C treating it as (a) classical oscillator (b) Planck's oscillator.

Solution:→ (a) classical oscillator

$$E = KT = 1.4\times10^{-23}\times300$$

$$= 4.2\times10^{-21}\,J$$

(b)

$$E_{Plank} = \left(\frac{h\nu}{e^{h\nu/kT} - 1}\right) = \left[\frac{6.6\times10^{-34}\times4.2\times10^{12}}{e^{\frac{6.6\times10^{-34}\times4.2\times10^{12}}{1.4\times10^{-23}\times300}} - 1}\right] = 2.99\times10$$

7.b:→ Determine the probability of finding a particle trapped in a box of length L in the region from 0.45L to 0.55L for the ground state.

Solution:→ The eigen function of a particle trapped in box of length L is given by

$$\psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

The Probability

$$P = \int_{x_1}^{x_2} |\psi_n|^2 dx$$

$$P = \int_{x_1}^{x_2} \frac{2}{L} \sin^2\left(\frac{n\pi x}{L}\right) dx$$

$$P = \frac{2}{L} \int_{0.45L}^{0.55L} \frac{1}{2}\left(1 - \cos\frac{2n\pi x}{L}\right) dx$$

$$P = \frac{1}{L} \int_{0.45L}^{0.55L} \left(1 - \cos\frac{2n\pi x}{L}\right) dx$$

$$P = \frac{1}{L}\left[x - \frac{L}{2n\pi} \sin\frac{2n\pi x}{L}\right]_{0.45L}^{0.55L}$$

For ground state $n = 1$

$$P = \frac{1}{L}\left[\left\{0.55L - \frac{L}{2\pi}\sin(1.10\pi)\right\} - \left\{0.45L - \frac{L}{2\pi}\sin(0.90\pi)\right\}\right]$$

$$P = \left[\left\{0.55 - \frac{1}{2\pi}\sin(1.10\pi)\right\} - \left\{0.45 - \frac{1}{2\pi}\sin(0.90\pi)\right\}\right]$$

$$P = (0.55 - 0.45) - \frac{1}{2\pi}(\sin 198° - \sin 162°)$$

$$P = 0.10 - \frac{1}{2\pi}(-0.309 - 0.309)$$

$$P = 0.10 - \frac{1}{2\pi}(-0.618)$$

$$P = 0.10 + 0.0984$$

$$P = 0.1984$$

$$P = 19.84\%$$

Ques 8.(a) Find two lowest permissible energy state for an electron which is confined in one-dimensional infinite potential box of width 3.5×10^{-9} m.

Sol^n:- We know that energy of particle mass m is

$$E_n = \frac{n^2 h^2}{8ma^2}$$

$$E_n = \frac{n^2 (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (3.5 \times 10^{-9})^2}$$

$$= 0.49 \times 10^{-19} n^2 \text{ J}$$

or

$$E_n = \frac{0.49 \times 10^{-19} n^2}{1.6 \times 10^{-19}} \text{ eV} = 0.0306 n^2 \text{ eV}$$

For $n = 1$, $\boxed{E_1 = 0.0306 \text{ eV}}$

for $n = 2$, $E_2 = 0.0306 \times (2)^2$

$$\boxed{E_2 = 0.1224 \text{ EV}}$$

Ques.8(b) X-rays of wavelength 2Å are scattered from a black-body and X-rays are scattered at an angle of 45°. Calculate Compton shift, wavelength of scattered photon λ'.

Sol^n:- We know that compton shift,

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

$$\Delta\lambda = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^8}(1 - \cos 45)$$

$$= 0.243\left(1 - \frac{1}{\sqrt{2}}\right) \times 10^{-11}$$

$$\Delta\lambda = 0.007 \text{ Å}$$

$$\lambda' - \lambda = 0.007 \text{ Å}$$

$$\lambda' = 2 + 0.007 \rightarrow \boxed{\lambda' = 2.007 \text{ Å}}$$

Ques. 9 Calculate the wavelength of an α-particle accelerated through a potential difference of 200V.

Sol^n. The de-Broglie wavelength of an α-particle accelerated through a potential difference 'V' is given by

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

For α-particle

$$q = 2e = 2 \times 1.6 \times 10^{-19} \text{ Coulomb}$$

$$m = 4 \times \text{mass of proton} = 4 \times 1.67 \times 10^{-27} \text{ kg}$$

$$\therefore \lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 4 \times 1.67 \times 10^{-27} \times 2 \times 1.6 \times 10^{-19} \times 200}}$$

$$\lambda = 7.16 \times 10^{-13} m$$

$$\text{or } \boxed{\lambda = 7.16 \times 10^{-3} \text{Å} = 0.00716 \text{Å}}$$

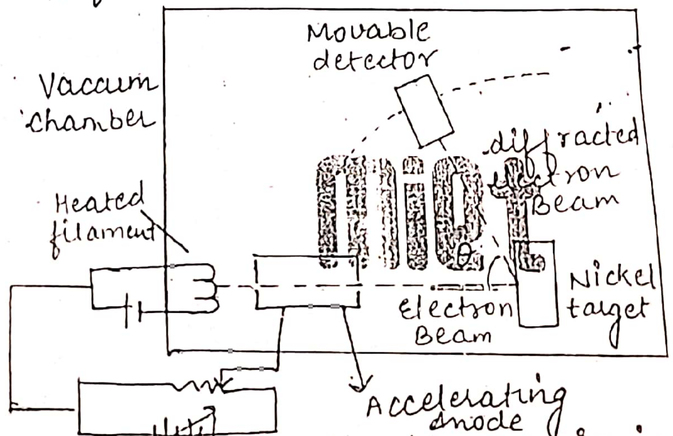
Q10: Describe the experiment of Davisson and Germer to demonstrate the wave character of electrons.

Sol^n

Davisson and Germer Experiment :-

It is the first experimental evidence of Material particle was predicted in 1927 by clinton Davisson and Lester Germer. This experiment not only confirm the existence of waves associated with electron by detecting de-broglie waves but also succeed in Measuring the wavelength.

Set up of davisson-Germer Experiment :-

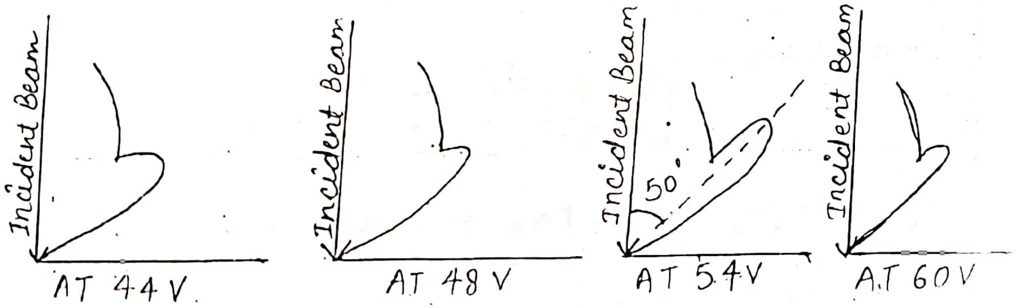


The electrons are produced by thermionic Emission f a tungsten filament F mounted in an electron g The ejected electrons are accelerated towards anode an electric field of known potential difference an collimated into a Narrow Beam. The whole arra used to emit Electrons and to accelerate and to foc called Electron Gun.

The Narrow Beam of Electron is allowed to fall Normally the surface of a Nickel crystal Because their atoms arranged in a regular pattern/Lattice so the surfa lattice of the crystal acts as a diffraction gratin and electrons are diffracted by crystals in different direction. The electrons scattered from target are collected by detector c which is also connected to a Galvanometer G and can be Moved along a circular scale s.

The electron are scattered in all direction by atoms in a crystal. The intensity of Electrons scattered in a particular direction is formed by using a detector on rotating the detector, the intensity of scattered beams can be Measured for different value of angle between Incident and scattered direction of Electron beam.

The various Graphs are plotted between scattering angle and intensity of scattered beam at different accelerating voltage.



Salient features observed from the graph:-

1) Intensity of scattered Electron depends upon angle of scattering.
2) A kink begins to appear in curve at 44 volts
3) This kink moves upward as the voltage increases and become More prominent for 54 Volts at 50°
4) The size of kink starts decreasing with further increase in accelerating voltage and drops almost to zero at 68 volts.
5) The kink at 54 volts offer evidence for existence of electron waves.

The crystal surface acts like a diffraction grating with spacing 'd'. The principal Maxima for such a grating must satisfy Bragg's Equation

where d is the interplanar spacing $d = 0.91\,Å$

$n = 1$ for first order

According to Davisson Germer experiment

$\theta + 2\phi = 180$

$\phi = 90 - \frac{\theta}{2}$

for $\theta = 50$; $\phi = 90 - 25$
$= 65$

Now; $2d\sin\phi = \lambda$

$2 \times 0.91 \times 10^{-10} \sin 65 = \lambda$

$\lambda = 1.65\,Å$

Now for electron having KE of 54 eV

$\lambda = \frac{h}{\sqrt{2mE}}$

$= \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 54 \times 1.6 \times 10^{-19}}}$

$\lambda = 1.67\,Å$

UNIT-2

UNIT : 2

Ques 1: What is the equation of continuity? Obtain the required expression for it. Also, give its physical significan

Ans: Electric current through surface area is given as

$$I = \oint \vec{J}\cdot d\vec{s} \quad \text{——} (1) \qquad (\vec{J} = \text{current densit}$$

Consider, closed surface 'S' enclosing volume 'V'. If 'ρ' is volum density then,

$$I = -\frac{dq}{dt} = -\frac{d}{dt}\int \rho \, dV \quad \text{——} (2)$$

According to law of conservation of charge, eq (1) and (2) will be equal.

$$\oint \vec{J}\cdot d\vec{s} = -\frac{d}{dt}\int \rho \, dV \quad \text{——} (3)$$

Applying Gauss Divergence theorem,

$$\int (\vec{\nabla}\cdot\vec{J})\, dV = -\frac{d}{dt}\int \rho \, dV = -\int \frac{\partial}{\partial t}\rho \, dV$$

$$\int \left(\vec{\nabla}\cdot\vec{J} + \frac{\partial \rho}{\partial t}\right) dV = 0$$

for an arbitary volume, integrand must be zero, then

$$\boxed{\vec{\nabla}\cdot\vec{J} + \frac{\partial \rho}{\partial t} = 0} \longrightarrow \text{Equation of continuity}$$

Physical Significance : Law of conservation of charg

For steady state, $\frac{\partial \rho}{\partial t} = 0$

$$\therefore \boxed{\vec{\nabla}\cdot\vec{J} = 0} \text{—— Equation of contin for 'steady stat}$$

Ques 2: Explain the concept of displacement current and show how it leads to modification of Ampere law.

Ans: The concept of displacement current was introdu to resolve the paradox of changing capacitor.

-Current $\xrightarrow{\text{produces}}$] Magnetic field.
Also, changing Electric field $\xrightarrow{\text{produces}}$]

It means that changing electric field is equivalent to current which flows as long as electric field is changing. This equivalent current in vacuum or dielectric produces same magnetic field as the conduction current and is known as displacement current (I_d)

$$I_d = \varepsilon_0 \frac{d\phi_e}{dt}$$

Modification of Ampere's law :-
Differential form of Ampere's law,

$$\vec{\nabla} \times \vec{H} = \vec{J} \quad \text{—— (1)} \quad \rightarrow \text{for steady state current.}$$

For time varying field, current density should be modified.
Taking divergence of (1),

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{H}) = \vec{\nabla}\cdot\vec{J}$$

$$\vec{\nabla}\cdot\vec{J} = 0 \quad \text{—— (2)} \quad [\because \vec{\nabla}\cdot(\vec{\nabla}\times\vec{H}) = 0]$$

But according to equation of continuity, $\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t}$ —— (3)

Equation (2) & (3) contradict each other, thus Maxwell suggested that something should be added to $\vec{J}$, such that eqn (1) become,

$$\vec{\nabla}\times\vec{H} = \vec{J} + \vec{J'} \quad \text{—— (4)}$$

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{H}) - \vec{\nabla}\cdot(\vec{J}+\vec{J'})$$

$$\vec{\nabla}\cdot\vec{J} = -\vec{\nabla}\cdot\vec{J'}$$

or $\vec{\nabla}\cdot\vec{J'} = \partial\rho/\partial t \quad [\because \vec{\nabla}\cdot\vec{J} = -\partial\rho/\partial t]$

$$\vec{\nabla}\cdot\vec{J'} = \frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{D}) \quad [\because \vec{\nabla}\cdot\vec{D} = \rho]$$

$$\vec{\nabla}\cdot\vec{J'} = \vec{\nabla}\cdot\left(\frac{\partial}{\partial t}\vec{D}\right)$$

or $\boxed{\vec{J'} = \frac{\partial \vec{D}}{\partial t}}$

Thus, $\boxed{\vec{\nabla}\times\vec{H} = \vec{J} + \underbrace{\partial\vec{D}/\partial t}_{\text{displacement current density}}}$ — Modified Ampere's law.

Ques.3: Derive Maxwell's equations in differential form. Give physical significance of each equation. Describe why monopole does not exist?

Ans:- Maxwell's equations in differential form:

(1) According to Gauss law in electrostatics:

$$\oint \vec{E}\cdot d\vec{s} = \frac{q}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int \rho\, dv$$

$$\oint \varepsilon_0 \vec{E}\cdot d\vec{s} = \int \rho\, dv$$

$$\oint \vec{D}\cdot d\vec{s} = \int \rho\, dv \quad \text{—— (1)} \quad [\because \vec{D} = \varepsilon_0\vec{E}]$$

electric displacement vector

Applying Gauss divergence theorem,

$$\oint \vec{D}\cdot d\vec{s} = \int(\vec{\nabla}\cdot\vec{D})\, dv$$

$$\int(\vec{\nabla}\cdot\vec{D})\, dv = \int \rho\, dv$$

$$\int(\vec{\nabla}\cdot\vec{D} - \rho)\, dv = 0$$

for an arbitrary volume, integrand must be zero.

$$\vec{\nabla}\cdot\vec{D} - \rho = 0$$

$$\boxed{\vec{\nabla}\cdot\vec{D} = \rho} \rightarrow \text{Maxwell first eq}^n$$

(2) According to Gauss law in magnetostatics

$$\oint \vec{B}\cdot d\vec{s} = 0$$

using Gauss divergence theorem

$$\oint \vec{B}\cdot d\vec{s} = \int \vec{\nabla}\cdot\vec{B}\, dV = 0$$

or $\boxed{\vec{\nabla}\cdot\vec{B} = 0}$ [$\because$ arbitrary volume]
↳ Maxwell's second eq^n.

(3). According to Faraday's law of electromagnetic induction emf $= -\frac{d\phi_B}{dt}$, we know that emf $= \oint \vec{E}\cdot d\vec{l}$

then $\oint \vec{E}\cdot d\vec{l} = -\frac{d\phi_B}{dt} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{s} \quad [\because \phi_B = \int \vec{B}\cdot d\vec{s}]$

magnetic flux

So $\oint \vec{E}\cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$

using Stoke's theorem on above left side

then : $\int (\vec{\nabla}\times\vec{E})\, d\vec{s} = -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$

$\Rightarrow \int (\vec{\nabla}\times\vec{E} + \frac{\partial \vec{B}}{\partial t})\, d\vec{s} = 0$

$\Rightarrow \vec{\nabla}\times\vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$ [for arbitrary surface]

$\Rightarrow$ or $\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}}$ → Maxwell's third eqn.

(4) According to Ampere's circuital law,

$\oint \vec{H}\cdot d\vec{l} = I$

$\Rightarrow \oint \vec{H}\cdot d\vec{l} = \int \vec{J}\cdot d\vec{s}$ [$\because I = \int \vec{J}\cdot d\vec{s}$]

$\Rightarrow \int (\vec{\nabla}\times\vec{H})\, d\vec{s} = \int \vec{J}\cdot d\vec{s}$ [Using Stoke's theorem]

$\Rightarrow \int (\vec{\nabla}\times\vec{H} - \vec{J})\cdot d\vec{s} = 0$

$\Rightarrow \vec{\nabla}\times\vec{H} - \vec{J} = 0$ [For an arbitrary surface]

$\Rightarrow \vec{\nabla}\times\vec{H} = \vec{J}$ ——— (1)

→ Conduction current density

eqn(1) is valid for steady state but for varying fields it should be modified.

Taking divergence of eqn(1) we get

$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{H}) = \vec{\nabla}\cdot\vec{J}$

or $\vec{\nabla}\cdot\vec{J} = 0$ [Divergence of curl = 0]

But by equation of continuity

$\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t}$ ——— (2)

So, Maxwell realized this contradiction and suggested that total current density in eqn(1)

$\vec{\nabla}\times\vec{H} = \vec{J} + \vec{J}'$ ——— (3)

Taking divergence of eqn(3)

we get $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{H}) = \vec{\nabla}\cdot\vec{J} + \vec{\nabla}\cdot\vec{J}'$

$\vec{\nabla}\cdot\vec{J} = -\vec{\nabla}\cdot\vec{J}'$ [$\because \vec{\nabla}\cdot(\vec{\nabla}\times\vec{H}) =$

$\vec{\nabla}\cdot\vec{J}' = \frac{\partial \rho}{\partial t}$ [$\because \vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t}$

$\vec{\nabla}\cdot\vec{J}' = \frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{D})$

or $\vec{\nabla}\cdot\vec{J}' = \vec{\nabla}\left(\frac{\partial \vec{D}}{\partial t}\right)$

or $\vec{J}' = \frac{\partial \vec{D}}{\partial t}$

So, eqn (3) will be $\boxed{\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$ → Maxwell's f equation

** Physical significance of Maxwell's equations:-

(1). $\oint \vec{E}\cdot d\vec{s} = \frac{1}{\varepsilon_0}\int \rho\, dv$ → It is Gauss law in electrostatics. It states that the surface integral of Electric field over any closed surface is equal to $\frac{1}{\varepsilon_0}$ times of net charge enclosed by that surface.

(2). $\oint \vec{B}\cdot d\vec{s} = 0$ → Gauss law in magnetostatics. It states that the net magnetic flux through any closed surface area is zero. It means magnetic monopole does not exist.

(3). $\underbrace{\oint \vec{E}\cdot d\vec{l}}_{\text{Induced emf around any closed Path}} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{s}$ → Faraday law of electromagnetic induction.

** negative rate of change of magnetic flux bounded by surface w.r.t. time.

(4). $\oint \vec{H}\cdot d\vec{l} = \vec{J} + \varepsilon\frac{d}{dt}\int \vec{E}\cdot d\vec{s}$ → Modified Amper's law

** $\because$ Varying electric field produces magnetic fi

Ques. 4 (a): State and deduce Poynting theorem for the flow of energy in an electromagnetic field. Also discuss its physical significance.

Ans: Poynting' theorem describes the flow of energy or power in an electromagnetic field during propagation of uniform plane wave.

Derivation: Maxwell's third and fourth equations

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{—— (1)} ; \quad \vec{\nabla}\times\vec{H} = \vec{J}+\frac{\partial \vec{D}}{\partial t} \quad \text{—— (2)}$$

Taking dot product of eq^n (1) with $\vec{H}$ and eq^n (2) with $\vec{E}$

$$\vec{H}\cdot(\vec{\nabla}\times\vec{E}) = -\vec{H}\cdot\frac{\partial \vec{B}}{\partial t} \quad \text{—— (3)}$$

$$\vec{E}\cdot(\vec{\nabla}\times\vec{H}) = \vec{E}\cdot\left(\vec{J}+\frac{\partial \vec{D}}{\partial t}\right) \quad \text{—— (4)}$$

Subtract (4) from (3)

$$\underbrace{\vec{H}\cdot(\vec{\nabla}\times\vec{E}) - \vec{E}\cdot(\vec{\nabla}\times\vec{H})}_{\text{vector identity}} = -\vec{H}\cdot\frac{\partial \vec{B}}{\partial t} - \vec{E}\cdot\vec{J} - \vec{E}\cdot\frac{\partial \vec{D}}{\partial t}$$

$$\left[\ \vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \vec{H}\cdot(\vec{\nabla}\times\vec{E}) - \vec{E}\cdot(\vec{\nabla}\times\vec{H})\ \right]$$

$$\therefore\ \vec{\nabla}\cdot(\vec{E}\times\vec{H}) = -\vec{H}\cdot\frac{\partial \vec{B}}{\partial t} - \vec{E}\cdot\vec{J} - \vec{E}\cdot\frac{\partial \vec{D}}{\partial t}$$

$$\text{or}\quad -\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \vec{H}\cdot\frac{\partial \vec{B}}{\partial t} + \vec{E}\cdot\frac{\partial \vec{D}}{\partial t} + \vec{E}\cdot\vec{J} \quad \text{—— (5)}$$

Now $\vec{D} = \varepsilon\vec{E}$ and $\vec{B} = \mu\vec{H}$

$$\vec{E}\cdot\frac{\partial \vec{D}}{\partial t} = \varepsilon\vec{E}\cdot\frac{\partial \vec{E}}{\partial t} = \frac{1}{2}\varepsilon\frac{\partial}{\partial t}(\vec{E}\cdot\vec{E}) = \frac{\partial}{\partial t}\left[\frac{1}{2}\varepsilon E^2\right]$$

Similarly $\vec{H}\cdot\frac{\partial \vec{B}}{\partial t} = \frac{\partial}{\partial t}\left[\frac{1}{2}\mu H^2\right]$

From eq^n (5) $-\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \vec{E}\cdot\vec{J} + \frac{\partial}{\partial t}\left[\frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2\right]$

Taking volume integration of above eq^n

$$-\int_V \vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \int_V \vec{E}\cdot\vec{J}\,dV + \frac{d}{dt}\int_V \frac{1}{2}(\varepsilon E^2 + \mu H^2)\,dV$$

Applying Gauss divergence theorem on L.H.S., we get

$$\underbrace{\int_V(\vec{E}\cdot\vec{J})\,dV}_{(I)} = \underbrace{\frac{d}{dt}\int_V\frac{1}{2}[\varepsilon E^2+\mu H^2]\,dV}_{(II)} - \underbrace{\oint_S(\vec{E}\times\vec{H})\cdot d\vec{S}}_{(III)}$$

Physical Significance

** Represent work energy theorem and is called Poynting theorem.

** I^st term represent the total power dissipated in a volume V.

** II^nd term → The sum of energy stored in electromagnetic field.

** III^rd term → The rate at which the energy is carried out of volume V, across its boundary surface by electromagnetic wave.

** Poynting theorem is based on conservation of energy.

(b) What do you mean by Poynting vector? Write its dimension and unit also

Ans. The amount of field energy passing through unit area of the surface per unit time is called Poynting vector. It is represented by

$$\vec{S} = \vec{E}\times\vec{H}$$

$$\boxed{S = EH} \qquad [\because \vec{S} = EH\sin 90^\circ = EH]$$

* It is along the direction of wave propagation

** Unit : Joule / m² sec. or Watt/m².

Dimension : $[S] = \frac{\text{Energy}}{\text{Area}\ \text{Time}}$ $[ML^2T^{-2}]$ / $[L^2][T]$

$[S] = [M T^{-3}]$

Ques5: Derive the electromagnetic wave equations in free space. Prove that the velocity of plane electromagnetic wave in free space is given by $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$.

Ans. Maxwell's equation in free space

$$\vec{\nabla}\cdot\vec{E} = 0 \quad \text{——— (1)}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \quad \text{——— (2)}$$

$$\vec{\nabla}\times\vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \quad \text{——— (3)}$$

$$\vec{\nabla}\times\vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \text{——— (4)}$$

Taking curl of eq^n (3),

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\vec{\nabla}\times\left(\mu_0 \frac{\partial \vec{H}}{\partial t}\right)$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\mu_0 \frac{\partial}{\partial t}(\vec{\nabla}\times\vec{H})$$

[Due to vector identity and from eq^n(4)] $\rightarrow$ $\vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E} = -\mu_0 \frac{\partial}{\partial t}\left(\varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$

$$-\nabla^2\vec{E} = -\mu_0\varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \qquad \{\because \vec{\nabla}\cdot\vec{E} = 0$$

$$\boxed{\nabla^2\vec{E} = \mu_0\varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}} \quad \text{——— (5)}$$

Similarly $\boxed{\nabla^2\vec{H} = \mu_0\varepsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2}}$ ——— (6)

We know that the general wave equation is

$$\nabla^2\psi = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} \quad \text{——— (7)}$$

On comparing eq^n (5), (6) & (7)

$$v = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{1}{\sqrt{4\pi\times10^{-7}\times 8.85\times10^{-12}}}$$

$$= 2.99\times10^{8}\ m/s \approx c \text{ (Speed of light)}$$

Thus, the EM waves written as

$$\boxed{\nabla^2\vec{E} = \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}} \ \& \ \boxed{\nabla^2\vec{H} = \frac{1}{c^2}\frac{\partial^2 \vec{H}}{\partial t^2}}$$

Ques6: Show that electric and magnetic vectors are no[rmal] to the direction of propagation of electromagnetic wave (Transverse N[ature]).

Ans: We know that $\nabla^2\vec{E} = \mu_0\varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$; $\nabla^2\vec{H} = \mu_0\varepsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2}$

Solution of above eq^{n}s

$$\vec{E}(r,t) = \underbrace{\vec{E}_0}_{\text{amplitude}} e^{i(\vec{K}\cdot\vec{r}-\omega t)} \ ; \ \vec{H}(r,t) = H_0 e^{i(\vec{K}\cdot\vec{r}\ldots}$$

$\vec{K}$ (propogation vector) $= \underbrace{K}_{\text{magnitude of vector } \vec{K}}\hat{n} = \frac{2\pi}{\lambda}\hat{n} = \frac{2\pi\nu}{c}\hat{n} = \frac{\omega}{c}\hat{n}$ ($\hat{n}$ ↓ unit vector in direction of propagation)

Now, $\nabla = \frac{\partial}{\partial r}$ [$\because$ no angular coordinates in $\vec{E}$ and $\vec{H}$]

Now, $\frac{\partial}{\partial r}\vec{E}(r,t) = \frac{\partial}{\partial r}\vec{E}_0 e^{i(\vec{K}\cdot\vec{r}-\omega t)} = i K \vec{E}_0 e^{i(\vec{K}\cdot\vec{r}-\omega t)}$

or $\frac{\partial}{\partial r}\vec{E}(r,t) = iK\vec{E}(r,t) \Rightarrow \boxed{\frac{\partial}{\partial r} = iK = \nabla}$

Also, $\frac{\partial}{\partial t}\vec{E}(r,t) = \frac{\partial}{\partial t}\vec{E}_0 e^{i(\vec{K}\cdot\vec{r}-\omega t)} = -i\omega\vec{E}(r,t)$

$$\boxed{\frac{\partial}{\partial t} = -i\omega} \quad \text{——— (2)}$$

Using (1) in eq^n $\vec{\nabla}\cdot\vec{E} = 0$ and $\vec{\nabla}\cdot\vec{H} = 0$. So,

$i\vec{K}\cdot\vec{E} = 0 \Rightarrow \vec{K}\cdot\vec{E} = 0$ or $\boxed{\vec{K}\perp\vec{E}}$ — (3)

and $i\vec{K}\cdot\vec{H} = 0 \Rightarrow \vec{K}\cdot\vec{H} = 0$ or $\boxed{\vec{K}\perp\vec{H}}$ — (4) [$\because$ K...

Using (1) & (2) in eq^n $\vec{\nabla}\times\vec{E} = -\mu_0\frac{\partial \vec{H}}{\partial t}$. So,

$$i(\vec{K}\times\vec{E}) = \mu_0 i\omega\vec{H}$$

$$K(\hat{n}\times\vec{E}) = \mu_0\omega\vec{H}$$

$$\vec{H} = \frac{K}{\mu_0\omega}(\hat{n}\times\vec{E}) \Rightarrow \boxed{\vec{H} = \frac{1}{\mu_0 c}(\hat{n}\times\vec{E})}$$

Thus, eq^n (3), (4) and (5) shows that EM waves are transverse in nature.

Ques 1: What do you mean by depth of penetration and characteristics impedance? (skin depth)

Ans. Depth of Penetration :- When an EM wave propagate in a medium, its amplitude decreases inside the medium from the surface, which is known as Attenuation.

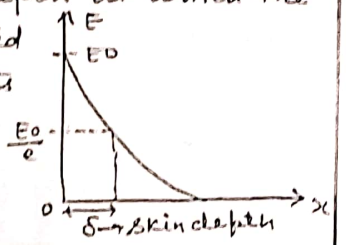
The amplitude of EM wave at depth 'x' in a medium

$$\boxed{E = E_0 e^{-\alpha x}}$$

α → attenuation constant

E_0 → amplitude of wave at surface

The depth of penetration is the depth at which the strength of electric field associated with EM wave reduces to $\frac{1}{e}$ to its initial value.



→ Characteristics impedance :

The ratio of magnitude of $\vec{E}$ to the magnitude of $\vec{H}$ is known as characteristics impedance (Z_0)

$$Z_0 = \left|\frac{\vec{E}}{\vec{H}}\right| = \frac{E}{H} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$= \sqrt{\frac{4\pi \times 10^{-7}}{8.854 \times 10^{-12}}}$$

$$\boxed{Z_0 = 376.72 \text{ ohms}}$$

↳ dimension of electric resistance

Ques 8: Calculate the magnitude of Poynting vector at the surface of Sun. Given that power radiated by Sun is 5.4×10^{23} W and its radius is 7×10^{8} m

Soln: Poynting vector, $S = \frac{\text{Power radiated}}{4\pi r^2}$

$$= \frac{5.4 \times 10^{23}}{4 \times 3.14 \times (7 \times 10^{8})^2} = 2.77 \times 10^{7} \frac{\text{Watt}}{m^2}$$ Ans

Ques 9: A 100 watt sodium lamp radiating its power. Calculate the electric and magnetic field strength at distance of 5m from lamp.

Soln.

$$S = \frac{P}{4\pi r^2} = \frac{100 \text{ W}}{4 \times 3.14 \times (5m)^2}$$

$$S = 3.185 \times 10^{-1} \frac{J}{m^2 s}$$

$\because \vec{S} = \vec{E} \times \vec{H} = EH \sin 90° = EH$

$\therefore EH = 3.185 \times 10^{-1} \frac{J}{m^2 s}$ —— (1)

Also, $\frac{E}{H} = 376.6\,\Omega$ —— (2)

Using (1) & (2), we get

$$E^2 = 119.95 \frac{V^2}{m^2}$$

or $\boxed{E = 10.95\, V/m}$

Now from eqn (2).

$$H = \frac{E}{376.6\,\Omega}$$

$$= \frac{10.95}{376.6}$$

$$\boxed{H = 0.0291 \frac{\text{Amp-turn}}{m}}$$

Question 10:-> The earth recive solar energy from the sun is 2 cal/cm²-min. what will be the peak values of electric field and magnetic field?

Solⁿ:-> Given $S = 2\ cal/cm^2 min$

$$= \frac{2 \times 4.2\ J}{(10^{-2} m)^2\ 60\ sec}$$

$$S = 1400\ J/m^2 sec$$

we know Poynting vecter

$$S = E \times H = EH \sin 90$$

$$S = EH$$

so

$$EH = 1400\ Joule/m^2 sec \quad \text{---} (1)$$

we know characteristic impedance of vacuum is

$$\frac{E}{H} = 376.77\ \Omega \quad \text{---} (2)$$

From eqⁿ (1) & (2)

$$E^2 = 527478 (N/C)^2$$

$$E = 726.27\ N/C$$

Putting this value in eqⁿ (1)

$$H = \frac{1400}{726.27} = 1.98\ N/A.m$$

we know

$$E_0 = E\sqrt{2}$$

$$E_0 = 726.27\sqrt{2}\ N/C$$

$$\boxed{E_0 = 1027.1\ N/C}$$

$$H_0 = H\sqrt{2}$$

$$= 1.93\sqrt{2}\ N/A.m$$

$$\boxed{H_0 = 2.73\ N/A.m}$$

UNIT-3

UNIT 3 : Wave Optics

Ques 1: Why two independent sources cannot produce interference? Write the main condition of sustained interference.

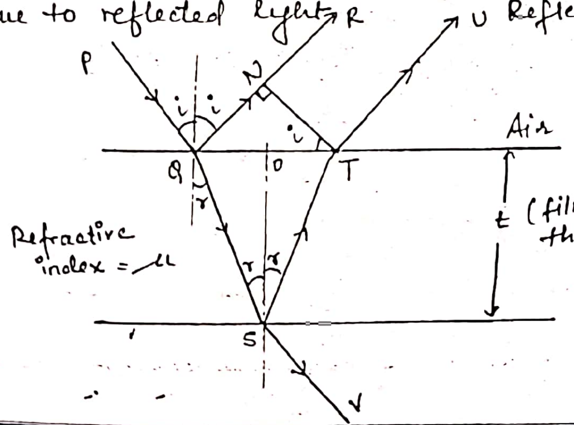
Ans: Two independent sources cannot produce interference because they are not coherent. The phase difference between the two waves will not be constant because light is emitted due to millions of atoms and their number goes on changing in a quite random manner.

→ Condition of sustained interference :–

(1) Two interferring waves should be coherent.
(2) Light source should be monochromatic.
(3) Two coherent sources must be narrow
(4) Separation between the coherent sources should be as small as possible.
(5) The distance of screen from two sources should be quite large.

Ques 2: Discuss the phenomenon of interference in thin film due to reflected light.

Ans:



Consider a film thickness 't' and refractive index μ (>1).

The optical path difference.

$$\Delta = \text{path } (QS+ST) \text{ in medium} - \text{Path } QN \text{ in air}$$

$$\Delta = \mu(QS+ST) - QN \quad \text{—— (1)}$$

In right angle ΔQSO, $\cos r = \frac{SO}{QS} \Rightarrow QS = \frac{t}{\cos r}$ —— (2)

Similarly in ΔSOT, $\cos r = \frac{SO}{ST} \Rightarrow ST = \frac{t}{\cos r}$ —— (3) $[SO = t]$

In ΔQTN, $\sin i = \frac{QN}{QT}$

$$QN = QT \sin i = (QO+OT)\sin i \quad \text{—— (4)}$$

Using (2), (3) and (4) in (1),

$$\Delta = \mu\left[\frac{t}{\cos r} + \frac{t}{\cos r}\right] - (QO+OT)\sin i$$

$$\Delta = \frac{2\mu t}{\cos r} - (QO+OT)\sin i \quad \text{—— (5)}$$

Now, ΔQSO & ΔSOT

$$\tan r = \frac{QO}{OS} \qquad \tan r = \frac{OT}{OS}$$

$$QO = OS \tan r \qquad OT = t\tan r \quad \text{—— (7)}$$

$$QO = t\tan r \quad \text{—— (6)}$$

Using (6) & (7) in (5).

$$\Delta = \frac{2\mu t}{\cos r} - (2t\tan r)\sin i$$

$$\Delta = \frac{2\mu t}{\cos r} - 2\mu t \tan r \sin r \qquad \left[\because \mu = \frac{\sin i}{\sin r}\right]$$

$$\Delta = \frac{2\mu t}{\cos r} - \frac{2\mu t \sin^2 r}{\cos r}$$

$$\Delta = \frac{2\mu t}{\cos r}(1-\sin^2 r) \Rightarrow \boxed{\Delta = 2\mu t\cos r}$$

According to Stoke's law, when light is reflected from the surface of optically denser medium, a path difference of $\lambda/2$ is introduced.

$$\therefore \boxed{\Delta = 2\mu t \cos r \pm \lambda/2}$$

Condition for maxima :-

$$\Delta = 2n\lambda/2$$

$$2\mu t \cos r + \lambda/2 = n\lambda$$

$$\boxed{2\mu t \cos r = (2n-1)\lambda/2} \quad ; n = 1, 2, 3 \ldots$$

Condition for minima :-

$$\Delta = (2n+1)\lambda/2$$

$$2\mu t \cos r + \frac{\lambda}{2} = (2n+1)\lambda/2$$

$$\boxed{2\mu t \cos r = n\lambda} \quad ; n = 0, 1, 2 \ldots$$

Ques 3: What are Newton's rings? Why the centre of Newton's rings is dark in reflected system? Show that diameter for bright rings are proportional to square root of odd natural number and for dark ring, diameter is proportional to square root of natural number.

Ans : Newton's rings are the phenomenon in which an interference pattern is created by the light's reflection between two surfaces. Alternate dark and bright fringes obtained.

The centre of Newton's ring is dark because at the point of contact, the path difference is zero, but one of the interfering ray is reflected, so effected path difference

becomes '$\lambda/2$', i.e. the condition of dark intensity is created. So, centre of Newton rings is dark

$$\Delta = 2\mu t \cos r + \lambda/2$$

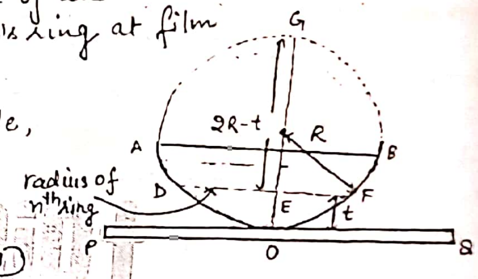
for normal incidence, $r = 0$ and at centre, $t = 0$

$$\therefore \boxed{\Delta = \lambda/2}$$

→ Diameter of Bright and Dark rings :-

Let: $R \rightarrow$ radius of curvature of lens

$r \rightarrow$ radius of n^{th} Newton's ring at film thickness 't'.



From the property of a circle,

$$DE \times EF = OE \times EG$$

$$r \times r = t(2R - t)$$

$$r^2 = 2Rt - t^2 \quad \text{--- (1)}$$

$$\because t << R, \quad \therefore r^2 = 2Rt$$

$$\Rightarrow \boxed{t = \frac{r^2}{2R}} \quad \text{--- (2)}$$

For bright rings :

Using eq (2) in eq^n $2\mu t + \frac{\lambda}{2} = 2n\frac{\lambda}{2}$

$$\frac{2\mu r^2}{2R} + \frac{\lambda}{2} = 2n\lambda/2$$

$$r^2 = \frac{(2n-1)\lambda R}{2\mu}$$

for nth ring,

$$r_n^2 = \frac{(2n-1)\lambda R}{2\mu}$$

$$\left(\frac{D_n}{2}\right)^2 = \frac{(2n-1)\lambda R}{2\mu} \qquad [\because r = D/2]$$

$$D_n^2 = \frac{2(2n-1)\lambda R}{\mu} \Rightarrow \boxed{D_n = \sqrt{\frac{2(2n-1)\lambda R}{\mu}}}$$

For air film, $\mu = 1$ $\therefore D_n = \sqrt{2(2n-1)\lambda R}$

$$\boxed{D_n = K\sqrt{(2n-1)}} \quad (4) \qquad \text{Let } \sqrt{2\lambda R} = K$$

For Dark rings:
Using eq (2) in

$$2\mu t = 2n\frac{\lambda}{2}$$

$$2\mu\frac{r^2}{2R} = n\lambda$$

$$r^2 = \frac{n\lambda R}{\mu}$$

for n^{th} ring, $r_n^2 = \frac{n\lambda R}{\mu}$

$$\left(\frac{D_n}{2}\right)^2 = \frac{n\lambda R}{\mu} \qquad \left[\because r = \frac{D}{2}\right]$$

$$D_n^2 = \frac{4n\lambda R}{\mu}$$

$$D_n = \sqrt{\frac{4n\lambda R}{\mu}}$$

for air film, $\mu = 1$

$$D_n = K\sqrt{n} \qquad [\because \sqrt{4\lambda R} = K]$$

$$\boxed{D_n \propto \sqrt{n}} \quad (5)$$

Thus, eq (4) and eq (5) shows that diameter of bright rings are proportional to the square root of odd natural number and for dark ring, diameter is proportional to square root of natural number.

Ques4: (a) Light of wavelength 6000 Å falls normally on a thin wedge shaped film of refractive index 1.4 forming the fringes that are 2mm apart. Find angle of wedge.

Soln: fringe width, $\beta = \frac{\lambda}{2\mu\theta}$

Given $\lambda = 6000\text{Å} = 6000 \times 10^{-10}\,m$, $\mu = 1.4$, $\beta = 2mm = 2\times10^{-3}\,m$

$$\therefore 2\times10^{-3} = \frac{6000\times10^{-10}}{2\times1.4\times\theta}$$

$$\theta = \frac{6000\times10^{-10}}{4\times1.4\times10^{-3}} = 1.07\times10^{-4}\text{ radian}$$

$\left(1 \text{ radian} = \frac{180^\circ}{\pi}\text{ degree}\right)$

$$\theta = 1.07\times10^{-4}\times\frac{180^\circ}{\pi}\text{ degree} \Rightarrow \boxed{\theta = 0.0061^\circ}$$

Q4 (b) In Newton's ring experiment, the diameter of 4^{th} and 12^{th} dark ring are 0.4 cm and 0.7 cm respectively. Deduce the diameter of 20^{th} dark ring.

Soln: If $D_{n+p} \rightarrow$ diameter of $(n+p)^{th}$ dark ring
$D_n \rightarrow$ diameter of n^{th} dark ring

$$\therefore D_{n+p}^2 - D_n^2 = 4p\lambda R \quad (1)$$

Given, $n = 4$, $n+p = 12$. so, $p = 8$

To find: D_{20}

$$D_{12}^2 - D_4^2 = 4(8)\lambda R$$

$$(0.7)^2 - (0.4)^2 = 32\lambda R$$

$$\boxed{\lambda R = \frac{0.33}{32}} \quad (2)$$

Now, for 20^{th} dark ring,

$$D_{20}^2 = 4n\lambda R$$

Here $n = 20$, $\lambda R = \frac{0.33}{32}$ (from eq (1))

$$\therefore D_{20}^2 = \frac{4\times20\times0.33}{32} = 0.825 \Rightarrow \boxed{D_{20} = 0.908\text{ cm}}$$

Q4(c) :- Calculate the thickness of soap bubble thin film that will result in constructive interference in reflected light. The film is illuminated with light of wavelength 5000Å and refractive index of film is 1.45.

Solⁿ : Condition of constructive interference.

$$2\mu t\cos r + \frac{\lambda}{2} = 2n\frac{\lambda}{2}$$

$$2\mu t\cos r = (2n-1)\frac{\lambda}{2}$$

For minimum thickness, $n=1$, $\cos r = 1$ $(r=0)$ for normal incidence

$$\therefore \quad t = \frac{\lambda}{4\mu}$$

$$t = \frac{5000\times10^{-10}\,m}{4\times1.45}$$

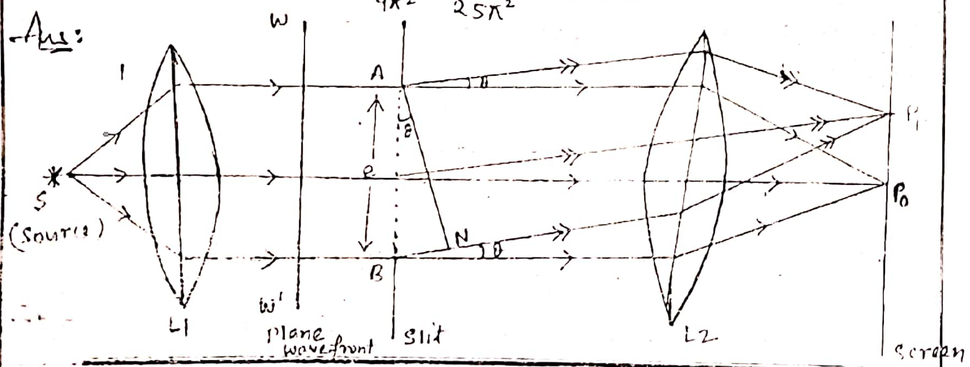
$$t = 0.86\times10^{-7}\,m$$

$$\boxed{t = 8.6\times10^{-8}\,m}$$

Ques5 : Discuss the phenomena of Fraunhofer's diffraction at a slit and show that relative intensities of the successive maximas are nearly

$$1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \ldots\ldots$$

Ans:



A monochromatic beam of light of wavelength λ incident on lens L_1 and emerging light falls on slit AB. Diffracted light observed on screen.

Path difference $BN = e\sin\theta$ —(1)

Phase difference $= \frac{2\pi}{\lambda}e\sin\theta$

$$\phi = \frac{2\pi}{\lambda}e\sin\theta \quad \text{—(2)}$$

If slit divided into n small parts, so

$$\frac{\phi}{n} = \frac{2\pi}{n\lambda}e\sin\theta$$

$$\delta = \frac{2\pi}{n\lambda}e\sin\theta \quad \text{—(3)}$$

According to composition of n simple harmonic motion of equal amplitude 'a' and common phase difference, the resultant amplitude at P_1

$$R = \frac{a\sin n\delta/2}{\sin\delta/2} = \frac{a\sin n\frac{2\pi}{2n\lambda}e\sin\theta}{\sin\frac{2\pi}{2n\lambda}e\sin\theta}$$

$$R = \frac{a\sin\frac{\pi}{\lambda}e\sin\theta}{\sin\frac{\pi}{n\lambda}e\sin\theta}$$

$$R = \frac{a\sin\alpha}{\sin(\frac{\alpha}{n})} \quad \text{where } \alpha = \frac{\pi}{\lambda}e\sin\theta$$

If α is small, $\frac{\alpha}{n}$ is very small, so $\sin\frac{\alpha}{n} = (\frac{\alpha}{n})$

$$R = \frac{a\sin\alpha}{(\frac{\alpha}{n})} = \frac{na\sin\alpha}{\alpha},$$

$$\boxed{R = \frac{A\sin\alpha}{\alpha}} \quad \text{—(4) where } A = na$$

Intensity $I = R^2$

$$\boxed{I = \frac{A^2\sin^2\alpha}{\alpha^2}} \quad \text{—— (5)}$$

Positions of Maxima & Minima :→

From equation (4)

$$R = \frac{A\sin\alpha}{\alpha} = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{\lfloor 3} + \frac{\alpha^5}{\lfloor 5} - \frac{\alpha^7}{\lfloor 7} + \ldots\right]$$

$$R = A\left[1 - \frac{\alpha^2}{\lfloor 3} + \frac{\alpha^4}{\lfloor 5} - \ldots\right]$$

The value of R will be Maximum if $\alpha = 0$

$\alpha = 0$

$\frac{\pi}{\lambda} e \sin\theta = 0$, $\sin\theta = 0$, $\theta = 0$ — This is condⁿ for principle maxima.

Position of minima and secondary maxima :→ Differentiate eqⁿ (4) w.r.to α and equal to zero. i.e.

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\left[A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\right] = 0$$

$$A^2 \frac{2\sin\alpha}{\alpha}\left[\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right] = 0$$

either $\sin\alpha = 0$ or $\alpha\cos\alpha - \sin\alpha = 0$

$\sin\alpha = 0$

$\alpha = \pm n\pi$

$\frac{\pi}{\lambda} e\sin\theta = \pm n\pi$

$\boxed{e\sin\theta = \pm n\lambda}$

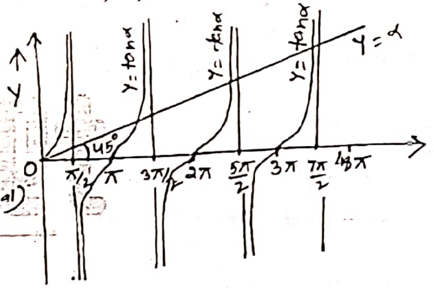
Condⁿ of minima

$\alpha\cos\alpha - \sin\alpha = 0$

$\alpha\cos\alpha = \sin\alpha$

$\alpha = \tan\alpha$ [Position of secondary maxima

$Y = \alpha$, $Y = \tan\alpha$



From graph, intersection point is maxima.

$\alpha = 0, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2} \ldots$

$\alpha = 0$ → Maxima (Principal)

$\alpha = \pm\frac{3\pi}{2}$

$\frac{\pi}{\lambda} e\sin\theta = (2n+1)\pi/2$

$\boxed{e\sin\theta = (2n+1)\frac{\lambda}{2}}$

Putting value of α in eqⁿ (5), we get

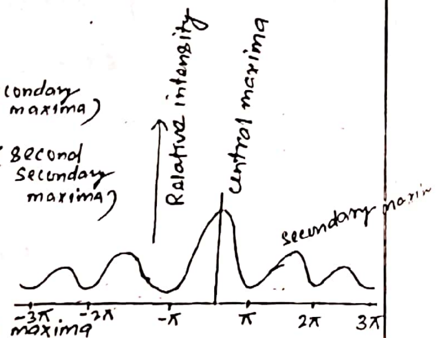
$I = A^2 = I_0$

$I_1 = A^2\left(\frac{\sin 3\pi/2}{3\pi/2}\right)^2 = A^2\frac{4}{9\pi^2} = \frac{I_0}{22}$ (secondary maxima)

$I_2 = A^2\left(\frac{\sin 5\pi/2}{5\pi/2}\right)^2 = A^2\frac{4}{25\pi^2} = \frac{I_0}{61}$ (second secondary maxima)

Thus

$I_0 : I_1 : I_2 \ldots = 1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} \ldots$



→ First secondary maxima is 4.5% of central maxima

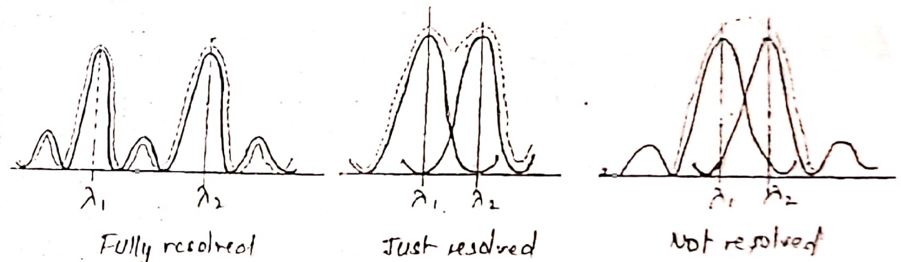
→ Second secondary maxima is 1.61% of central maxima.

Ques 6: What do you mean by resolving power of an optical instrument? State Rayleigh criterion of resolution.

Ans: The ability of an optical instrument to just resolve the images of two close point objects is called its resolving power.

$\boxed{\text{Resolving power (R.P.)} = \frac{\lambda}{d\lambda}}$ → $d\lambda$: smallest wavelength difference   or $\boxed{R.P. = nN}$ → n: order, N: no. of lines

Rayleigh criterion of Resolution: According to Rayleigh, two equally intense spectral lines are just resolved by an optical instrument when the central maxima of diffraction pattern due to one source falls exactly on first minima of diffraction pattern of other and vice-versa.



Ques 7: A diffraction grating used at normal incidence gives yellow line ($\lambda = 6000Å$) in certain spectral order superimposed on a blue line ($\lambda = 4800Å$) of next higher order. If the angle of diffraction is $\sin^{-1}(\frac{3}{4})$, then calculate the grating element.

Solⁿ: Principal maxima for λ_1, $(e+d)\sin\theta = n\lambda_1$ — (1)

Let nth order of λ_1 coincide with $(n+1)$th of λ_2

$\therefore$ $(e+d)\sin\theta = \underbrace{n\lambda_1 = (n+1)\lambda_2}$

$n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$ — (2)

Using ② in ①, $(e+d)\sin\theta = \frac{\lambda_2}{\lambda_1-\lambda_2}\lambda_1$ $\quad \because \theta = \sin^{-1}\frac{3}{4}$

$(e+d) = \frac{\lambda_2\lambda_1}{(\lambda_1-\lambda_2)\sin\theta}$ $\quad \sin\theta = \frac{3}{4}$

$= \frac{4800\times10^{-10}\times6000\times10^{-10}}{(6000-4800)\times10^{-10}\times3/4}$

$= \frac{48\times10^{-8}\times6000\times4}{1200\times3}$

$(e+d) = 32000\times10^{-10}m = 3.2\times10^{-6}m$ Ans

Q(?): A diffraction grating used at normal incidence gives green line ($\lambda = 5450Å$) in certain spectral order superimposed on violet line ($\lambda = 4100Å$) of next higher order. If the angle of diffraction is 30°, then how many lines per cm are there in a grating?

Soln: Given, n^{th} order $\rightarrow \lambda_1 = 5450Å$] superimposed
$(n+1)^{th}$ order $\rightarrow \lambda_2 = 4100Å$

for principal maxima, $(e+d)\sin\theta = n\lambda$.

So, $(e+d)\sin\theta = n\lambda_1 = (n+1)\lambda_2 \Rightarrow \boxed{n = \frac{\lambda_2}{\lambda_1-\lambda_2}}$

$\therefore (e+d)\sin\theta = n\lambda_1 = \frac{\lambda_2}{(\lambda_1-\lambda_2)}\lambda_1$

$(e+d) = \frac{5450\times10^{-10}\times4100\times10^{-10}}{(5450-4100)\times10^{-10}\times\sin30°}$ $\quad (\sin30° = 1/2)$

$= \frac{5450\times4100\times10^{-10}}{1350\times(1/2)}$

$\boxed{(e+d) = 33.10\times10^{-7}m}$ Ans

Number of lines per m $= \frac{1}{(e+d)} = \frac{1}{(33.10\times10^{-7})m} = 3.0\times10^{5}$

Number of lines per cm $= \frac{1}{(33.10\times10^{-7}\times10^{2})cm} = 3.0\times10^{3}$ Ans

Ques 9(i): A plane transmission grating has 15000 lines per Inches. Find the resolving power of grating and smallest wavelength difference that can be resolved with light of 6000Å in second order.

Soln: Given, $n = 2$, no. of lines per inch = 15000, $\lambda = 6000$

$\therefore$ Resolving power $= nN$
$= 2\times15000 = 30,000$ Ans

Also, Resolving power, $\frac{\lambda}{d\lambda} = nN = 30,000$

$d\lambda = \frac{6000\times10^{-8}cm}{30,000} = 0.2\times10^{-8}cm$

$d\lambda = \underline{0.20Å}$ Ans

Ques 9(ii) – A plane transmission grating has 16000 lines per an inch over a length of 5 inches. Find in the wavelength re of 6000Å in second order (1) resolving power of grating. (2) smallest wavelength difference that can be resolved.

Soln: Given, $\lambda = 6000Å$, $n = 2$
No. of lines per inches = 16000, length of grating = 5 inches
$\therefore$ total no. of lines on grating per inches $= 16000\times5$
i.e. $N = 80000$

(1) Resolving power $= nN = 2\times80,000 = 16\times10^{4}$ Ans

(2) Resolving power $= \frac{\lambda}{d\lambda}$

$16\times10^{4} = \frac{\lambda}{d\lambda}$

$d\lambda = \frac{6000\times10^{-8}cm}{16\times10^{4}} = 0.0375\times10^{-8}cm$

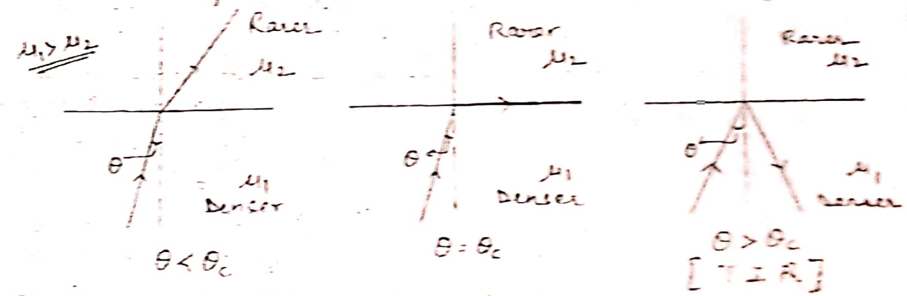
$d\lambda = 3.75\times10^{-10}cm$ Ans

UNIT-4

UNIT V : Fibre Optics and Laser

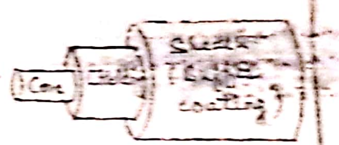
Ques 1 : What is the principle of optical fibre ? With the help of well labelled diagram, name the components of an optical fibre.

Ans: Principle of optical fibre :- Total Internal Reflection (TIR)



Structure : Consists of three parts

(a) Core : Innermost part of the fibre surrounded by second layer (cladding). The refractive index of core is slightly more than cladding.

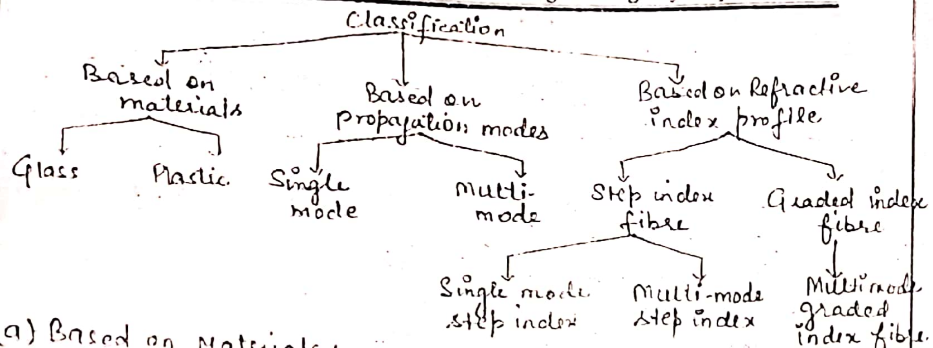


(b) Cladding : Optical properties different from core.

(c) Sheath : Outer layer of fibre which protect the fibre from crashing, contamination and moisture.

Ques 2 : What do you understand by an optical fibre and discuss its classifications.

Ans: Optical fibre is very thin and flexible medium. It is used for transportation of optical energy from one point to another. It is made up of glass or plastic (generally insulating) material.



(a) Based on Materials :-

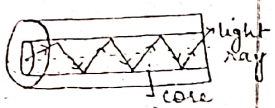
(1) Glass fibre : Made up of mixture of metal oxides and silica glasses.

(2) Plastic fibre : Made up of plastics.

(b) Based on Number of modes :-

(1) Single mode fibre (SMF) :-(i) Only single mode is transmitted through an optical fibre.

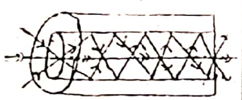
(ii) Smaller core diameter (5 um - 10 um) and high cladding diameter (70 um - 100 um)



(iii) Used for long distance communication like telephone lines.

(2) Multimode fibre (MMF) :- (i) When more than one mode is transmitted through an optical fibre.

(ii) Core diameter (20-150 um) and cladding diameter (~~40~~70-125 um)



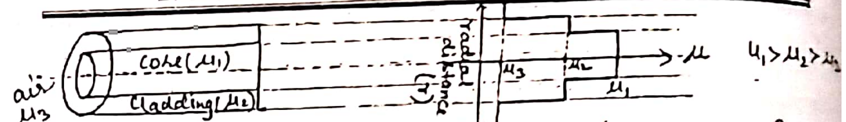
(iii) Not suitable for long distances communication.

(iv) Propagation of light is easy as compared to single mode fibre.

(c) Based on Refractive index :-

(1) Step index optical fibre : In step index fibre, the refractive index is uniform through core and cladding.

Step index → (i) Single mode step index fibre
→ (ii) Multi-mode step index fibre



(i) Single mode step index :- core diameter (5 to 10 um)
cladding diameter (50-125) um

(ii) Multimode step index :- core diameter (50-200) um
Cladding diameter (125-300) um

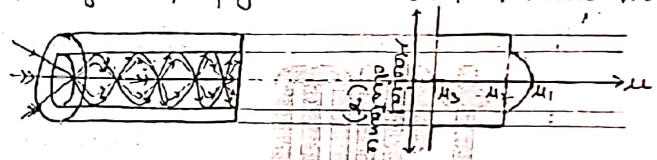
(2) Graded index optical fibre :

(i) Core has non-uniform refractive index that gradually decreases from the centre towards core-cladding interface.

(ii) Cladding has uniform refractive index.

(iii) Core diameter : 50 um ; cladding diameter : 70 um

(iv) Signal propagation : helical or skew manner.



Ques2(b) A communication system uses 25 km long fibre having a loss of 2.5 dB/km. The input power is 2500 uW. Compute output po[wer]

Sol^n: loss per km, dB/km $= \frac{-10}{L} \log_{10}\left(\frac{P_o}{P_i}\right)$

$$2.5 = \frac{-10}{25} \log_{10}\left(\frac{P_o}{2500\,\mu W}\right)$$

$$\text{antilog}\left(-\frac{2.5 \times 25}{10}\right) = \frac{P_o}{2500\,\mu W}$$

$$P_o = 2500\,\mu W \ \text{antilog}(-6.25)$$

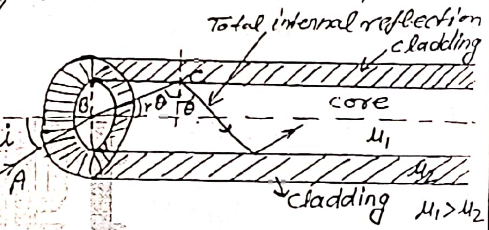
$$= 2500\,\mu W \times 5.62 \times 10^{-7}$$

$$P_o = \underline{0.0014\,\mu W} \quad \text{Ans}$$

Ques.3 What do you mean by critical angle, acceptance angle, acceptance cone and numerical aperture? Derive expression for them.

Ans. Critical angle: Critical angle is a particular incident angle for which refraction angle will be 90° in an optical rarer medium.

Acceptance angle:– Acceptance angle is defined as the maximum angle, that the incident light can make with the fibre axis, so that light ray will propagate through the fibre by total internal reflection within core.



* $\mu_1 \to$ Core refractive index
* $\mu_2 \to$ Cladding refractive index
* $\mu_1 > \mu_2$
* Incident angle, i
* refracted angle, r
* when angle i increase, r will increases and θ will be decreased.
* when $\theta > \theta_c$ (Critical angle) $\to$ Then total internal reflection will occure

using Snell's law at ray AB

$$\mu_0 \sin i = \mu_1 \sin r \quad \text{——— (1)}$$

from $\triangle BCD \to r = 180 - (90+\theta) = 90 - \theta$

or $\sin r = \sin(90-\theta) = \cos\theta$ ——— (2)

from (1) & (2) $\quad \mu_0 \sin i = \mu_1 \cos\theta$ ——— (3)

For $\theta \geq \theta_c \longrightarrow \sin i_{max} = \frac{\mu_1}{\mu_0} \cos\theta_c$ ——— (4)

$\left[\sin\theta_c = \frac{\mu_2}{\mu_1}; \text{(by critical angle)}\right]$ $\quad \sin i_{max} = \frac{\mu_1}{\mu_0}\sqrt{1-\sin^2\theta_c} = \frac{\mu_1}{\mu_0}\sqrt{1-\frac{\mu_2^2}{\mu_1^2}}$

$$i_{max} = \sin^{-1}\left(\frac{\sqrt{\mu_1^2-\mu_2^2}}{\mu_0}\right) = \sin^{-1}\left(\sqrt{\mu_1^2-\mu_2^2}\right)$$

or $\mu_0 = 1$ for air $\quad \boxed{i_{max} = \sin^{-1}\left(\sqrt{\mu_1^2-\mu_2^2}\right)}$

Acceptance cone:– If all possible direction of acceptance angle are considered at same time we get a cone corresponding to surface known as acceptance cone.

$\boxed{\text{Acceptance cone} - 2\, i_{max}}$

Numerical Aperture:
Numerical aperture (NA) determines the light gathering ability of the fibre. This is also called the merit of optical fibre.

$\boxed{NA = \sin i_{max}} \to NA = \sqrt{\mu_1^2-\mu_2^2}$ —(1)

We know that relative refractive index difference Δ;

$$\Delta = \frac{\mu_1-\mu_2}{\mu_1} = 1 - \frac{\mu_2}{\mu_1}; \quad \frac{\mu_2}{\mu_1} = 1-\Delta \text{ —(2)}$$

$$NA = \sqrt{\mu_1^2-\mu_2^2} = \mu_1\sqrt{1-\frac{\mu_2^2}{\mu_1^2}} \text{ ——— (3)}$$

Now from eq.(2) and (3), we get

$$NA = \mu_1\sqrt{[1-(1-\Delta)^2]} = \mu_1\sqrt{[2\Delta-\Delta^2]}$$

$\boxed{NA = \mu_1\sqrt{2\Delta}}$ $\quad [\Delta^2 \to \text{is very small}]$

Ques.4 A step index fibre has core refractive index 1.48, cladding refractive index 1.45. Compute acceptance angle, critical angle, and numerical aperture.

Soln:– (I) Numerical Aperture $NA = \sqrt{\mu_1^2-\mu_2^2} = \sqrt{(1.48)^2-(1.45)^2}$

$\boxed{NA = 0.24249}$

(II) Acceptance angle $\quad \sin i_{max} = NA$

$i_{max} = \sin^{-1}(NA)$

$= \sin^{-1}(0.24248)$

$\boxed{i_{max} = 14.03°}$

(III) Critical angle

$\sin\theta_c = \frac{\mu_2}{\mu_1} \to \theta_c = \sin^{-1}\left(\frac{1.46}{1.48}\right)$

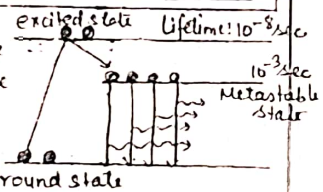
$\boxed{\theta_c = 80.?1°}$

Ques 5: Differentiate between spontaneous and stimulated emission of radiation. Which one is required for laser action? Also explain metastable state, population inversion and pumping.

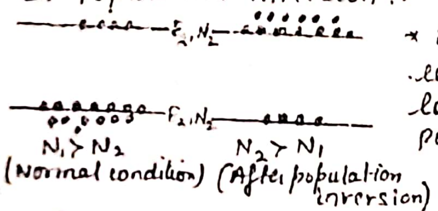
Ans

	Spontaneous Emission	Stimulated emission
1.	The emitted photons from various atoms have no phase relation	1. The emitted photon have same frequency and are in phase with incident photons.
2.	Emitted radiation are non-coherent.	2. Emitted radiation are coherent.
3.	Emitted photons can move in any direction.	3. for every incident photon, there are two outgoing photon moving in same direction.
4.	Rate of emission is proportional to number of atoms in excited state E_2.	4. Rate of emission is proportional to number of atoms in excited state E_2 and energy density of incident photon.
5.	It dis-favours laser action.	5. It favours laser action.

→ **Metastable state:** Excited state whose lifetime is about 10^{-3} second. Population inversion take place between metastable and lower state.



→ **Population inversion:-**



* For laser action to occur, the higher energy levels should be more populated than the lower energy levels, i.e, $N_2 > N_1$, called population inversion.

→ **Pumping:** It is an excitation mechanism which is required to raise the atoms or molecules into their excited state for laser action.

Ques 6: What are Einstein's coefficient? Establish a relation betw[een] them. Also discuss the essential conditions for laser action.

Ans: Einstein's coefficients are mathematical quantities which [are] a measure of the probability of absorption or emission of [light] by an atom.

→ Relation between them: Let $N_1 \to$ no. of atoms in state 1
$N_2 \to$ no. of atoms in state 2.

The rate of absorption (transition $1 \to 2$) is proportional to N[umber of atoms and] energy density $[u(\nu)]$, i.e,

$$P_{12} \propto N_1 u(\nu) \quad \text{or} \quad \boxed{P_{12} = B_{12} N_1 u(\nu)} \quad \text{—(1)}$$

B_{12} → Einstein coeffi[cient] of absorption.

The rate of spontaneous emission $(2 \to 1)$ is proportional to n[umber] of atoms in upper energy level (E_2), i.e

$$P_{21} \propto N_2 \quad \text{or} \quad \boxed{P_{21} = A_{21} N_2} \quad \text{—(2)}$$

A_{21} → Einstein coefficient [of] spontaneous

The rate of stimulated emission $(2 \to 1)$ is proportional to number o[f] atoms in E_2 & energy density $u(\nu)$, i.e,

$$P_{21} \propto N_2 u(\nu) \quad \text{or} \quad \boxed{P_{21} = B_{21} N_2 u(\nu)} \quad \text{—(4)}$$

B_{21} → Einstein coefficient of stim[ulated] emission

Now, total probability due to emission,

$$P_{21} = A_{21} N_2 + B_{21} N_2 u(\nu) \quad \text{—(5)}$$

At thermal equilibrium, $P_{12} = P_{21}$

$$N_1 B_{12} u(\nu) = N_2 A_{21} + B_{21} N_2 u(\nu)$$

$$u(\nu)[N_1 B_{12} - N_2 B_{21}] = N_2 A_{21}$$

$$u(\nu) = \frac{N_2 A_{21}}{N_1 B_{12} - N_2 B_{21}} = \frac{N_2 A_{21}}{\frac{N_2}{N_2}(N_1 B_{12} - N_2 B_{2[1]})}$$

$$= \frac{A_{21}}{\left(\frac{N_1}{N_2} B_{12} - B_{21}\right)} = \frac{A_{21}}{B_{21}\left(\frac{N_1}{N_2} \times \frac{B_{12}}{B_{21}} - 1\right)}$$

$$u(\nu) = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{N_1}{N_2} \frac{B_{12}}{B_{21}} - 1} \right] \quad \text{—(6)}$$

$\because B_{12} = B_{21}$

$$\therefore \quad u(\nu) = \frac{A_{21}}{B_{21}}\left[\frac{1}{\frac{N_1}{N_2}-1}\right] \quad \text{—— (7)}$$

According to Maxwell Boltzmann's distribution law,

$N_1 = N_0 e^{-E_1/kT}$ and $N_2 = N_0 e^{-E_2/kT}$

$$\therefore \frac{N_1}{N_2} = e^{(E_2-E_1)/kT}$$

$$\frac{N_1}{N_2} = e^{h\nu/kT} \quad \text{—— (8)} \qquad \because E_2 - E_1 = h\nu$$

Using (8) in (7),

$$\boxed{u(\nu) = \frac{A_{21}}{B_{21}} \frac{1}{e^{h\nu/kT}-1}} \quad \text{—— (9)}$$

According to Planck's radiation law,

$$\boxed{u(\nu) = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/kT}-1}} \quad \text{—— (10)}$$

On comparing (9) & (10),

$$\boxed{\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}} \rightarrow$$ Required relation between Einstein Coefficient

→ Essential conditions for laser action :—

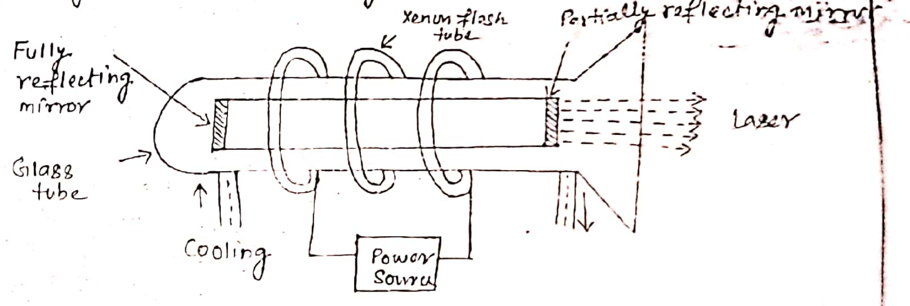
(1) Rate of emission must be greater than rate of absorption

(2) Probability of spontaneous emission must be negligible in comparision to probability of stimulated emission.

(3) The coherent beam of light must be sufficiently amplified.

Question:07 :→ Describe the principle and working of Ruby laser system. Compare it with He-Ne laser.

Ruby laser :→ (1) First solid laser developed by T H Maiman.
(2) It consist ruby cylindrical rod, ends are optically flat attached with partially and fully silvered mirror.
(3) Ruby rod is sorrounded by helical xenon flash tube.

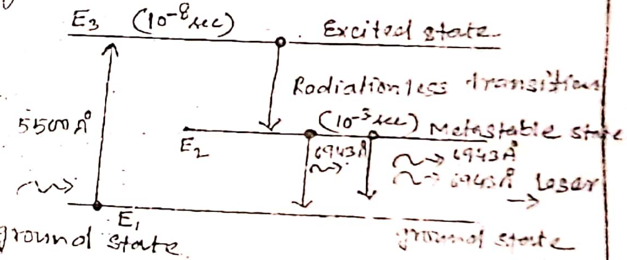


(4) Ruby rod is crystal of Al_2O_3 doped with 0.05% Cr_2O_3 so Al^{++} ions are replaced by Cr^{3+}.

(5) These Cr^{3+} ions gives pink colour to ruby.

(6) Ruby rod is 2 – 20 cm in length and 0.1 – 0.2 cm in diameter.

The energy level of Ruby laser is below.



→ E_1 is ground state.
→ E_2 is metastable state.
→ E_3 is excited state.
→ Principle :- Three level laser system.
→ Initially all atoms in ground state.
→ When pumping applying (5500Å), Atoms reached E_3 state, which have life time for staying 10^{-8} sec.
→ $E_3 \rightarrow E_2$ transition is radiation loss.
→ In metastable state, atom stay 10^{-3} sec. After 10^{-3} sec, atoms move toward downward and transfer our own energy to nearest atom. Population inversion occur $E_2 \rightarrow E_1$ and get laser (6943Å)

Ruby laser over He-Ne laser :-

→ Ruby laser produced a pulsed laser beam while He-Ne laser produced continuous laser beam.

→ Ruby laser is three level laser while He-Ne laser is four level laser

→ Ruby laser is solid laser while He-Ne laser is gas laser.

→ Optical pumping used in Ruby laser while electrical pumping used in He-Ne laser.

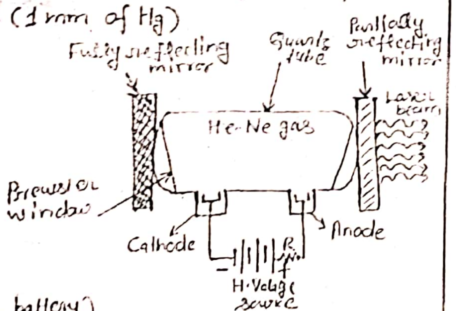
Ques 8: Illustrate the construction and working of He-Ne laser? Discuss important application of LASER.

Ans. Helium-Neon Gas Laser (Four Level Transition Laser)

Principle :- It is first gas laser which based on 'four Level System'.

Construction:

* Active medium: Gas mixture [He-Ne (5:1)]; total pressure about 1 torr (1 mm of Hg)



* Resonant Cavity:
Active medium enclosed between set of mirror one is fully reflecting and other is partially reflecting called Resonant cavity (Optical cavity)

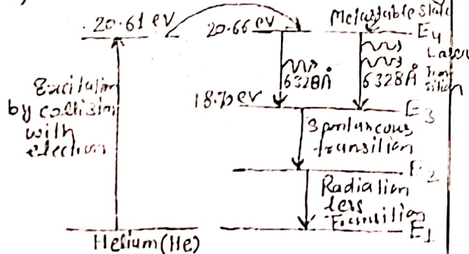
* Pumping Source:-
A powerful generator (High voltage battery) is used for electric discharge in tube.

* Active lasing atom → Ne-atom; He-atom → for selective pumping

Working:-

* Due to electrical discharge method the electrons collide with He + Ne-atoms and excite them.

* Because He-atoms are lighter than Ne-atom so it excite.



easily and goes to upper most excited state (20.61 eV) and Ne goes to 20.66 eV respectively.

* Due to collision of He-Ne atoms → help to achieving population inversion havier Ne-atoms.

* When Ne-atoms drop down → emits a photon (6328 Å). from (20.66 eV → 18.70 eV)

* Due to back & forth motion of the photon (6328 Å) from reflector it stimulates other excited Ne-atoms and a fresh 6328 Å photon emits in phase with first photon.

* The stimulated transition from 20.66 eV — 18.70 eV is the laser transition. The two photons will knock out more photons and process is repeated again and again photon multiplies.

* When gain is achieved, a portion of it passes through partially silvered end. Which is laser output.

Characteristics of He-Ne Laser:-

* → It produces a continuous laser beam.

* → It is four level laser system

* → Electric discharge method is used for pumping.

* → It has active medium in gaseous state.

* → Cooling arrangement is not required

* → It emits light of wavelength 6328 Å.

Application of Laser:-

(1). Industrial → (a) Drilling (b) Cutting (c) Welding

(2). Medicine → (a) In treatment of eye related problem
(b) In treatment of skin related problem
(c) In treatment of dentistry
(d) In treatment of Cancer etc.

(3) In optical communication:— Information sending through optical fibre.

(4) Atmospheric Studies

(5) Holography and LIDAR

Ques 9: (a) Calculate the population ratio of two states in He-Ne laser that produces light of wavelength 6000 Å at 300 K.

Soln: Given; $\lambda = 6000\,\text{Å} = 6000\times10^{-10}\,m$, $T = 300\,K$

$$\frac{N_2}{N_1} = e^{-(E_2-E_1)/kT}$$

$$\frac{N_2}{N_1} = e^{-\frac{hc}{\lambda kT}} \qquad \left[\because E_2-E_1 = h\nu = \frac{hc}{\lambda}\right]$$

$$= e^{-\frac{6.62\times10^{-34}\times3\times10^{8}}{6000\times10^{-10}\times1.38\times10^{-23}\times300}}$$

$$\boxed{\frac{N_2}{N_1} = e^{-80}}$$

Ques 9: (b) In Ruby laser, total numbers of Cr^{+3} ions is 2.8×10^{29}. If laser emits radiation of wavelength 7000 Å, then calculate energy of laser pulse.

Soln:

$$E = n h\nu = n\frac{hc}{\lambda}$$

$$= \frac{2.8\times10^{29}\times6.62\times10^{-34}\times3\times10^{8}}{7000\times10^{-10}}\ J$$

$$\boxed{E = 7.94\ J}$$

Ques 10: What do you understand by attenuation, dispersion and losses in an optical fibre?

Ans. Attenuation of optical signal in Fibre:

* When signals guided through an optical fibre, the reduction in amplitude or intensity of light signal is called attenuation or total loss.

* The losses are measured in (dB) and attenuation in (dB/km)

$$\boxed{\alpha = -\frac{10}{L}\log_{10}\left(\frac{P_o}{P_i}\right)\ dB/km}$$

$\alpha \rightarrow$ Attenuation coefficient

Attenuation Losses:

(a) Absorption loss (b) Scattering loss (c) waveguide loss

(a) Absorption Loss:- It is two types basically.

(I) Imperfections in the atomic structure of fibre material.

* Absorption due to presence of missing molecules or oxygen deffects. And also by the deffusion of Hydrogen molecules in the fibre material.

(II) Intrinsic absorption:

* It happens due to impurities in the fibre material.

* For pure fibre material → No intrinsic absorption.

(III) Extrinsic absorption:

* It happens due to impurities into fibre material during fibre such as iron, nickel and chromium during fabrication.

* Extrinsic absorption is caused by the electronic transition of these metal ions from one energy state to other.

(b) Scattering Loss:-

* This is second main reason of signal attenuation.

* It is caused due to presence of chemical impurities in the fibre which act as scattering centers for light waves.

* Rayligh scattering loss is inversely proportional to the fourth power of the wavelength (λ) of incident light.

$$\boxed{\text{Scattering loss} \propto \frac{1}{\lambda^4}}$$

* It means greater wavelength less scattering and vice versa.

* For communication → Avoided shorter wavelengths region such as UV and Visible range.

(C) Waveguide loss:

(I) Macro-bending loss:- Due to large curvature of fibre cable.
(II) Micro-bending loss: Due to small curvature or bending in fibre cable.
(III) Non uniform core radius.
(IV) Connecting issues in fibres.
(V) Axial distortion

Dispersion in optical fibre

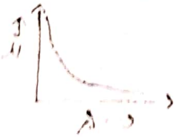
(I) Intermodal Dispersion or Multimode dispersion
* The broadening of output signals at the end point of the fibre of different modes (guided waves)

[Intermodal dispersion]

(II) Intramodal dispersion or material dispersion:
* It causes due to material of fibre.
* The refractive index is different for different wavelength.

$$\boxed{\mu = \frac{B}{\lambda} + \frac{C}{\lambda^2} + \frac{D}{\lambda^4} \approx \frac{B}{\lambda}}$$

& $\boxed{V\ (\text{velocity}) = \frac{c}{\mu}}$



* It means, shorter wavelength travel slower than longer wavelength.

(III) Waveguide dispersion:
* It arises due to fibre design like core radius etc.
* This dispersion limits the bandwidth of optical signal

$$V = \frac{2\pi a}{\lambda_0}\sqrt{n_1^2 - n_2^2} = \frac{2\pi a}{\lambda_0}(NA)$$

$a \to$ radius of core ; $\lambda_0 \to$ operating wavelength
$n_1 \to$ core refractive index , $n_2 \to$ cladding R.I.

* If $V < 2.405$ → support single mode fibre
If $V > 2.405$ → support multimode fibre

UNIT-5

Unit-V

Superconductors & Nanomaterials

Q.No1a→ What is superconductivity ?

Ans → Superconductivity is the property of some material that offer zero resistance when they are cooled below critical temperature.

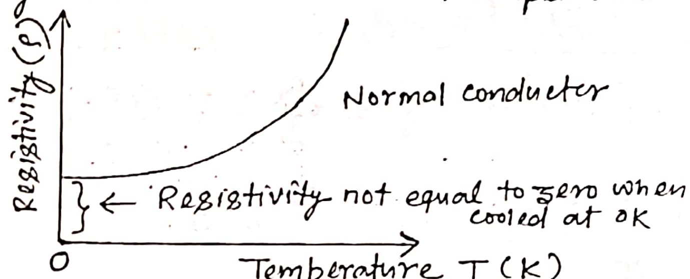
Q.No1b→ Discuss the temperature dependance of resistivity in superconducting materials.

Ans :→ Resistivity of materials depends on the temperature

$$\rho_T = \rho_0 (1+\alpha \Delta T)$$

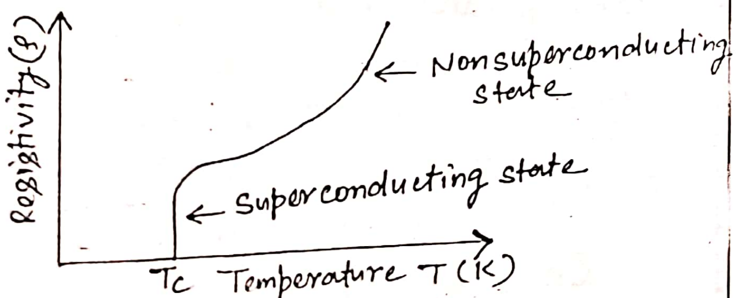
→ Resistivity of Normal conducter never will be zero when they are cooled below critical temperature.

For Normal conducter



→ Resistivity of superconducter or superconducting material will be zero when they are cooled below critical temperature (T_c).

For Superconducter



Q.No2→ Discuss Meissner effect.

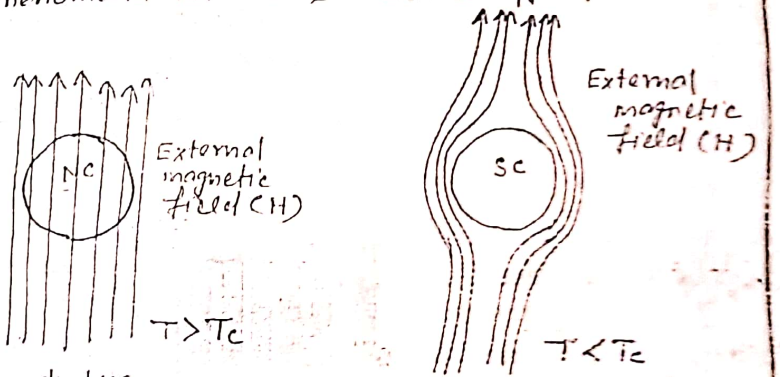
or

Show superconducters are perfect diamagnetism.

or

Show that perfect diamagnetism & zero resistivity are two independent.

Ans :→ When Normal conducter ($T>T_c$) are placed inside magnetic field, magnetic field lines penetrate them, but when superconducters ($T<T_c$) are placed inside the magnetic field, magnetic field lines do not penetrate them. This phenomena known as Meissner effect.



For superconducter

$$B = \mu_0 (H+M) \quad \text{---(1)}$$

Here $B = 0$

$$0 = \mu_0 (H+M)$$

$$H = -M \quad \text{---(2)}$$

we know, magnetic susceptibility

$$\chi = \frac{M}{H} \quad \text{---(3)}$$

Compare eqn (2) & (3)

$$\boxed{\chi = -1}$$

This show that superconductors are perfect diamagnetism

According to Maxwell's eqn

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{---(4)}$$

From ohm's law

$$V = IR, \quad \vec{E} = \frac{V}{d} \Rightarrow \frac{IR}{d}$$

$$\vec{E} = \vec{J}\frac{AR}{d}$$

$$\vec{E} = \vec{J}\rho \quad \text{where } \rho = \frac{AR}{d}$$

For superconducter $\rho = 0$

$$\text{So } \vec{E} = 0 \quad \text{---(5)}$$

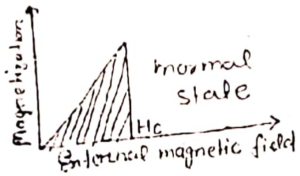
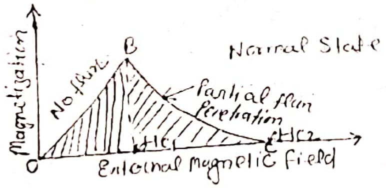
From eqn (4) & (5)

$$\frac{\partial \vec{B}}{\partial t} = 0 \Rightarrow B = \text{Constant}$$

It contradict to Meissner effect.

Q.3.a Describe type I and type II superconductors.

Ans.

Type-I Superconductors	Type-II Superconductors
1. Type-I Superconductors exhibit complete Meissner effect.	1. Type-II Superconductors show complete Meissner effect below H_{c1} and allow the flux to penetrate the S.C. between H_{c1} and H_{c2}, this state is mixed state, also called Vortex state.
2. Above critical field H_c, the S.C. becomes normal conductor.	2. Above H_{c2}, it comes in normal state.
3. Type-I S.C. are soft superconductors.	3. Type-II S.C are known as hard superconductors.
4. The critical field H_c is relatively low about 100 to 1000 gauss.	4. The value of H_{c2} is very large. Due to which, it posses high magnetic field and high current.
5. Type-I S.C → Al, Zn, Ga, etc	5. Type-II S.C are alloys like lead-indium alloy etc.
	

Q.3 b Why are type I Superconductors poor current carrying conductors?

Ans. Type-I Superconductors posses very low critical magnetic field, critical current and critical temperature, due to these property it behave as poor conductors.

Ques4a Explain the properties and application of superconductors.

Solⁿ. Properties:- (i) It is a low temp. phenomenon.

(ii) The transition temp. is different for different substa

(iii) Below the transition temp. the specific heat curve discontinous

(iv) Below the transition temp. the magnetic flux lin are rejected out of the superconductors.

(v) These are perfectly diamagnetic materials.

Application:- (i) For making superconducting mag

(ii) Formation of SQUIDS (Superconducting quantum interference device)

(iii) For progress of computer technology.

(iv) Formation of fast acting switches.

Ques4b What are the high temp. superconductors (HTS).

Solⁿ. The substances which demostate superconductivity temp. greater than 25 K. or their critical tem is higher than 25K are called HTS. they are basically copper oxides containing elements. Also they are Hard superconductors or Type-II.

Ex. Mercury - Barium - Calcium - copper - oxide

$$[Mg - Ba_2 - Ca_2 - Cu_3 - O_{10}]$$

Numericals

Q1) The transition temperature for Pb is 7.2K. However at 5K it loses the superconducting property if subjected to a magnetic field of 3.3×10^4 A/m. Find the value of $H_0(0)$ which will allow the metal to retain its superconductivity at 0K.

Solution $T_c = 7.2K$

$T = 5K$

$H_c(T=5K) = 3.3\times10^4$ A/m

$H_c(0) = ?$

We know that $H_c(T) = H_c(0)\left[1-\frac{T^2}{T_c^2}\right]$ i.e $H_c(0) = \frac{H_c(T)}{\left[1-\frac{T^2}{T_c^2}\right]}$

i.e $H_c(0) = \frac{3.3\times10^4}{\left[1-\frac{5^2}{(7.2)^2}\right]} = \frac{3.3\times10^4}{\left[1-\frac{25}{51.84}\right]} = 6.37\times10^4$ A/m

$\boxed{H_c(0) = 6.37\times10^4 \text{ A/m}}$ Ans

Q2) The transition temperature for lead is 7.26K. The maximum critical field for the material is 8×10^5 A/m lead has to be used as a superconductor subjected to magnetic field of 4×10^4 A/m. What precaution will have to be taken?

Solution $T_c = 7.26K$, $H_c(0) = 8\times10^5$ A/m, $H_c = 4\times10^4$ A/m

We know that $H_c(T) = H_c(0)\left[1-\frac{T^2}{T_c^2}\right]$ i.e $1-\frac{T^2}{T_c^2} = \frac{H_c(T)}{H_c(0)}$

$\frac{T^2}{T_c^2} = 1-\frac{H_c(T)}{H_c(0)}$ i.e $\boxed{T = T_c\left[1-\frac{H_c(T)}{H_c(0)}\right]^{1/2}}$

$T = 7.26\left[1-\frac{4\times10^4}{8\times10^5}\right]^{1/2} = 7.26\left[1-\frac{1}{20}\right]^{1/2} = 7.08K$

i.e $\boxed{T = 7.08K}$

The temperature of metal should be held below 7.08 K

Q3) At what temperature is $H_c(T) = 0.1 H_c(0)$ for lead (Pb) having $T_c = 7.2k$

Solution Using Formula $H_c(T) = H_c(0)\left[1-\frac{T^2}{T_c^2}\right]$

$0.1 H_c(0) = H_c(0)\left[1-\frac{T^2}{(7.2)^2}\right]$

$0.1 = \left[1-\frac{T^2}{51.84}\right]$

$\frac{T^2}{51.84} = 0.9$

$T^2 = 51.84\times0.9 = 46.656$

$T = (46.656)^{1/2}$

$\boxed{T = 6.83K}$ Ans

Q4) A superconducting material has a critical temperature of 3.7K in zero magnetic field of 0.306 tesla at 0K. Find the critical field at 2K.

Solution $H_c(T) = H_c(0)\left[1-\frac{T^2}{T_c^2}\right]$

$H_c(T) = 0.0306\left[1-\left(\frac{2}{3.7}\right)^2\right]$

$H_c(T) = 0.0306\times[1-0.292]$

$\boxed{H_c(T) = 0.0217 \text{ Tesla}}$ Ans

Q5) The critical field for niobium is 1×10^4 A/m at 8K and 2×10^5 A/m at 0K. Calculate the transition temperature of the

Solution $H_c = 1\times10^4$ A/m at $T = 8K$

$H_c(0) = 2\times10^5$ A/m

$T_c = ?$

$T_c = \frac{T}{\left[1-\frac{H_c(T)}{H_c(0)}\right]^{1/2}} = \frac{8}{\left[1-\frac{10^4}{2\times10^5}\right]^{1/2}} = \frac{8}{\left[1-\frac{1}{20}\right]^{1/2}} = \frac{8}{\left[\frac{19}{20}\right]^{1/2}} = 7.08K$

Q)If For a Specimen of superconductor, the critical field are 1.4×10^5 and 4.2×10^5 A/m respectively for temperature 14K and 13K respectively. Calculate the transition temperature and critical field at 0K & 4.2K

Solution $H_{c1} = 1.4\times10^5 A/m$, $T_1 = 14K$

$H_{c2} = 4.2\times10^5 A/m$, $T_2 = 13K$

$T_c = ?$, $H_c(0) = ?$, $H_c(T=4.2K) = ?$

Using formula $H_c(T) = H_c(0)\left[1-\frac{T}{T_c}\right]^2$

$H_{c1} = H_0\left[1-\left(\frac{14}{T_c}\right)^2\right] = 1.4\times10^5$ —(1)

$H_{c2} = H_c(0)\left[1-\left(\frac{13}{T_c}\right)^2\right] = 4.2\times10^5$ —(2)

Eq (2) / Eq (1)

$$\frac{\left[1-\left(\frac{13}{T_c}\right)^2\right]}{\left[1-\left(\frac{14}{T_c}\right)^2\right]} = \frac{4.2\times10^5}{1.4\times10^5}$$

$$\frac{T_c^2-169}{T_c^2-196} = 3$$

$T_c^2 - 169 = 3T_c^2 - 588$

$2T_c^2 = 419$

$\boxed{T_c = 14.47K}$ Ans

putting the value of T_c in eq(1)

$H_c(0)\left[1-\left(\frac{14}{14.47}\right)^2\right] = 1.4\times10^5$

$\boxed{H_c(0) = \frac{1.4\times10^5}{\left[1-\left(\frac{14}{14.47}\right)^2\right]} = 20.67\times10^5 A/m}$ Ans

$H_c(4.2) = H_0\left[1-\left(\frac{4.2}{14.47}\right)^2\right]$

$H_c(4.2) = H_0\times0.916$

$H_c(4.2) = 20.67\times10^5\times0.916$

$\boxed{H_c(4.2K) = 18.9\times10^5 A/m}$ Ans

Ques Calculate the critical current which can flow through a long thin superconducting wire of diameter 10^{-3}m given $H_c = 7.9\times10^3$ A/m

Solution We know that $I_c = 2\pi r H_c = 2\times3.14\times\left(\frac{10^{-3}}{2}\right)\times(7.9\times10^3)$

$\boxed{I_c = 24.8 \text{ ampere}}$ Ans

Ques 6a what are nanomaterials

Ans: The materials whose size lies between range 1nm-100nm are nanomaterials

$\boxed{1nm = 10^{-9}m}$

Ques 6b Explain nanoscience & nanotechnology

Ans: ⊙ Nanoscience is the study of phenomena & manipulation of materials at atomic, molecular & macromolecular sc...

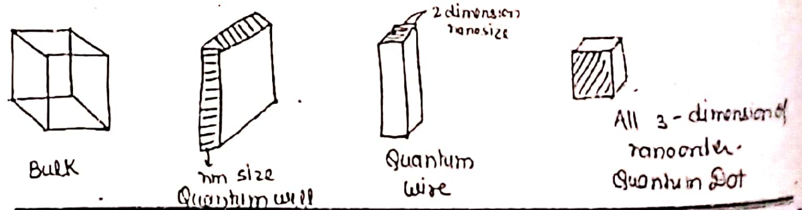
⊙ Nanotechnology is the technology of design, synthesis, characterization and application of material on nanosca...

Ques 7 What do you mean by quantum well, quantum wire & quantum dots.

Ans: Quantum well → If one dimension is reduced to the na... range while other dimension remain large, then the structure formed is known as quantum well.

Quantum wire → If two dimension are reduced to nanorar... while the third remains the same, then the structure so formed is called quantum wire.

Quantum Dots → When all the three dimensions of the mater... are reduced to nanorange, then it is called as quantum do... These are zero-dimensional nanostructures.

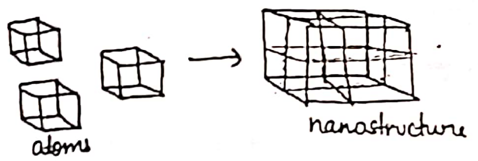


Q8 Write a short note on Bottom-up approach of nanomaterials synthesis. Also explain the Sol-gel method in detail.

or

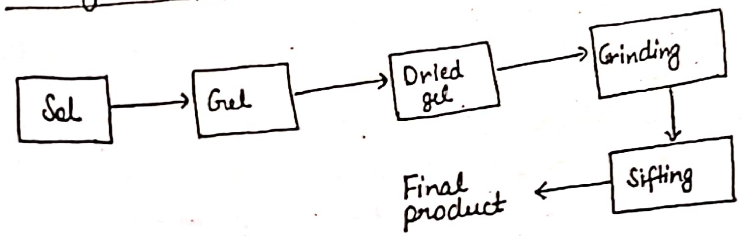
What are various steps involved in Sol-gel method ? On which Synthesis approach Sol-gel method is based?

Ans Bottom-up Technique is the approach in which materials are build up atom by atom or a technique to collect, consolidate and fashion individual atoms and molecule into the nano-structure.



Sol-gel method is a process used to create ceramic and glass materials in the form of powders, gels, or thin films. The method involves the hydrolysis and condensation of a precursor solution also known as sol to form a gel which is dried to form solid material. The precursor solution is made from metal alkoxides.

Working & steps involved in Sol-gel process



① A sol is a colloidal (the dispersed phase in which size of particles is so small that gravitational forces do not exist)

② A gel is a semi-rigid mass that forms when the solvent from the sol begins to evaporate and the particles left behind begin to join together in a continuous network. This is accomplished by sedimentation or centrifugation

③ The remaining liquid is removed using drying process accomplished by thermal treatment and remains left as dried gel. This further enhanced the mechanical stability

④ The dried gel is grinded to get the material into powdered form.

⑤ Once the powdere is obtained its lumps are removed using sifting process.

⑥ The final product obtained after sifting can be deposited on the suitable substrate to get ceramic or thin film.

→ This method allows for precise control of the composition a microstructure of the final material

→ This allows the production of materials in form of thin films.

Q9 Write a short note on Top-down approach, also explain Chemical Vapour deposition method (CVD).

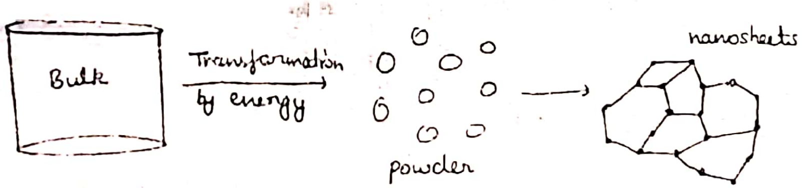
or

What is CVD process, explain the steps involved in CVD process using a labelled diagram?

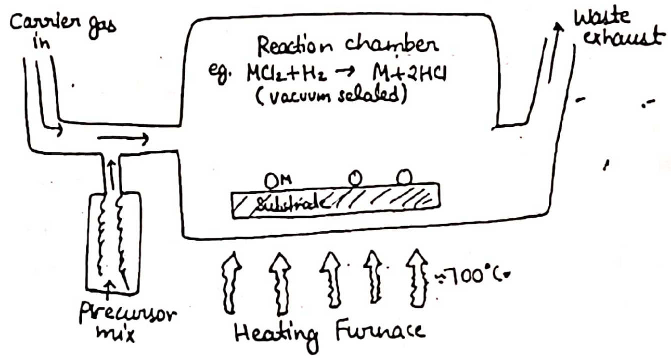
or

On which approach of nanomaterials fabrication/synthesis CVD method is based? Write a note on the CVD process.

Ans: Top-down approach is a technique in which materials & devices are synthesized or constructed by removing existing material from larger entities. Therefore in this technique a large scale object is gradually reduced in dimension to nanoscale pattern.



→ Chemical Vapour Deposition (CVD) is a process used to deposit thin films of various materials onto a substrate.
CVD is a versatile process that can be used to deposit a wide range of materials and is useful for a variety of applications including coatings, electronics, energy storage and more.



Steps involved in CVD process (working)

1. Transport of the reagents in gaseous phase often in carrier gas to the deposition zone.
2. Diffusion of gaseous precursors through the boundary layer
3. Adsorption of film precursors on to the growth surface.
4. Surface diffusion of precursors to growth surface.
5. Surface chemical reactions leading to deposition.
6. Desorption of by-products.
7. Transport of gaseous by products out of the reactor.

⇒ The CVD process takes place in a reactor chamber that is heated a high temperature. The substrate is placed in the chamber, a the precursors are introduced into the chamber in a contr manner. The precursors react with the substrate surface form a solid film. The thickness & properties of the film can be controlled by adjusting the temperature, pressure, flow rate of the precursors as well as the duration of the deposition proce
The various types of CVD including Low-pressure CVD, plasma-enha CVD, pulsed CVD, photo CVD, metal-organic CVD, etc.

Ques 10 Describe the Properties and various application of nanomaterial

Ans * Properties of Nanoparticles

1. Mechanical properties → very small nanoparticles having higher hardness. As grain size decreasing hardness increasing.
2. optical properties → clusters of different sizes have different Energy level separations. so their absorption is different for different clusters hence the different colours.
3. Magnetic properties → The smaller particles are more magnetic than bulk material.
4. Electronic properties → Energy levels of nanoparticles are discrete
5. Nanomaterials have a relatively larger surface to volume ratio than their bulk counter part.

* Applications of Nano Materials

1. Electronics → Most of Electronics devices based on the nanotechnology for Example formation of chips.
2. Diagnostics → Nanotechnology is helpful in medical diagnostics also.
3. optics → Nanoscience has Entered in the field of ligh emission by the use of light emitting diode (LED)
4. Sensors → sense based on nanotechnology are more sensitive & hence more effective.
5. Novel drugs → It aids in delivery of just the right amount of medicine to the exact spots of the body that need it.
6. Energy → Mono & poly crystalline silicon are widely used in solar cells.